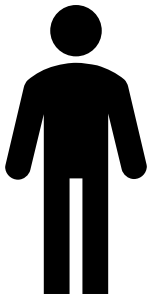
Ordered Context Effects

Short Slides Website

Xi Zhi (RC) Lim
Shanghai Jiao Tong University

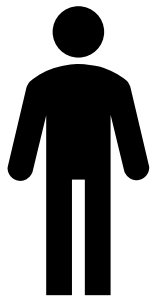


A

B

C

u



A

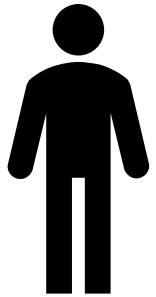
$$\max_{p \in A} \sum_{x} p(x)\mathrm{u}(x)$$

B

C

A

u



B

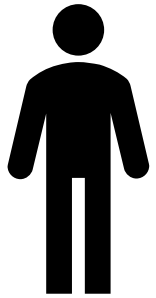
$$\max_{p \in B} \sum_{x} p(x)\mathrm{u}(x)$$

C

A

B

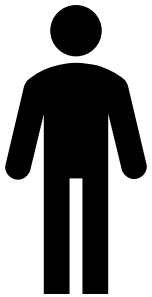
u



C

$$\max_{p \in C} \sum_{x} p(x)\mathrm{u}(x)$$

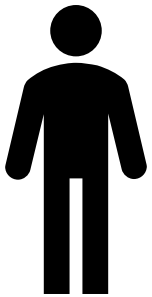
Expected Utility?



A $\max_{p \in A} \sum_x p(x)\mathrm{u}(x)$

B $\max_{p \in B} \sum_x p(x)\mathrm{u}(x)$

C $\max_{p \in C} \sum_x p(x)\mathrm{u}(x)$

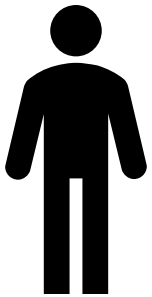


A u

B u

C u

EU functionals

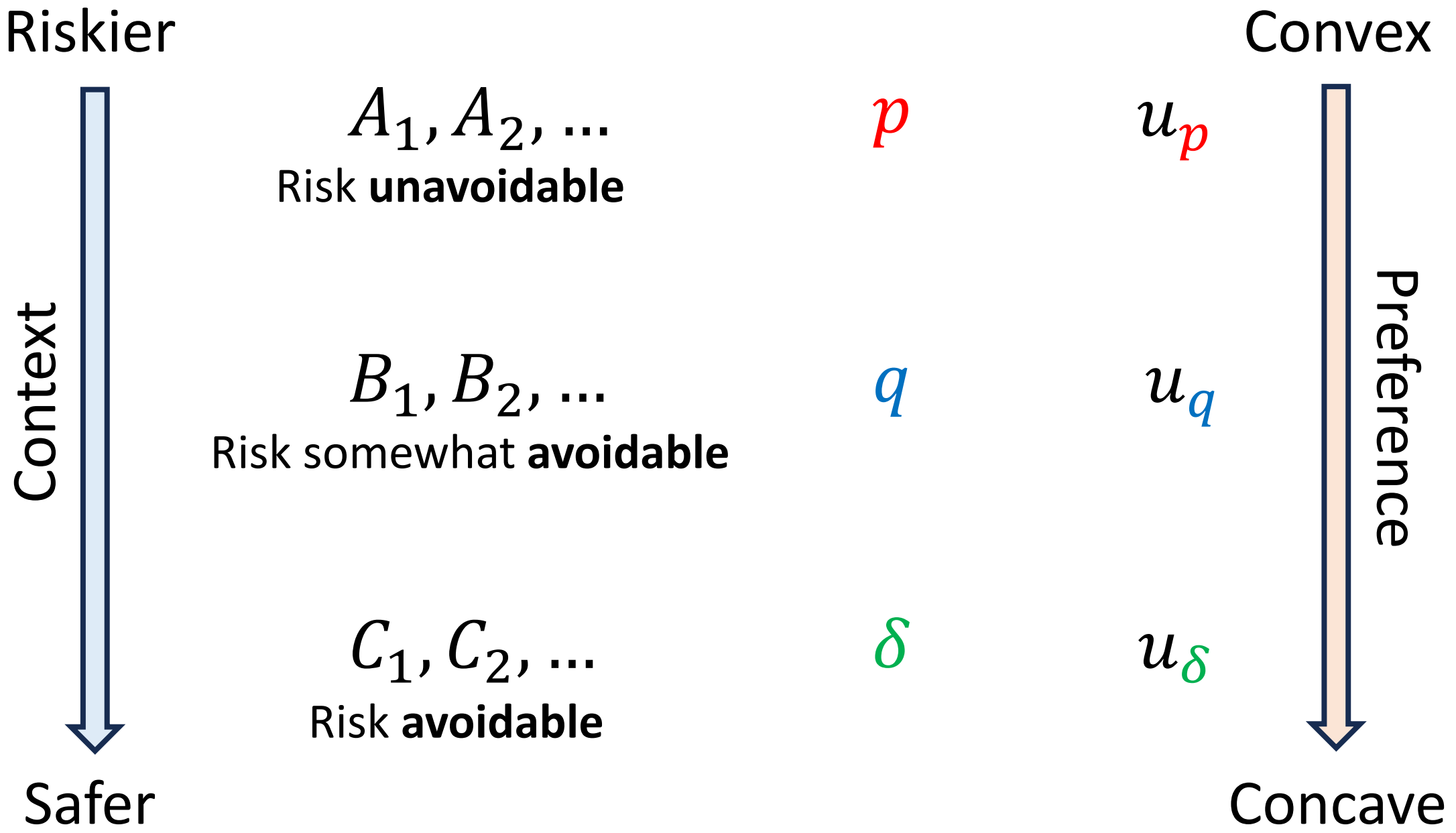


Context-dependent Preferences

$$A \quad \max_{p \in A} \sum_{x} p(x)\mathrm{u}(x)$$

$$B \quad \max_{p \in B} \sum_{x} p(x)\mathrm{u}(x)$$

$$C \quad \max_{p \in C} \sum_{x} p(x)\mathrm{u}(x)$$

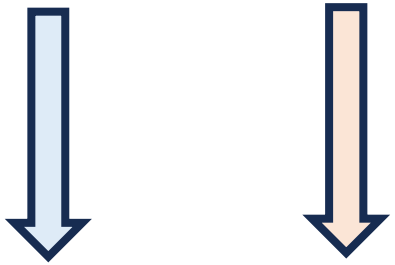

Riskier
Convex
Context
Preference
$A_1, A_2, \ldots$
Risk **unavoidable**
p
u_p
$B_1, B_2, \ldots$
Risk somewhat **avoidable**
q
u_q
$C_1, C_2, \ldots$
Risk **avoidable**
δ
u_δ
Safer
Concave

- Observation 1: Independence can fail
- Observation 2: Multiple objective functions ⇒ WARP can fail
- Observation 3: Same reference point ⇒ Independence & WARP hold
- **Q1: New behavior?**
- **Q2: Independence and WARP related?**
- **Q3: Functional forms and general utility representation related?**

Risk

$$\sum_x p(x)u_{r(A)}(x)$$

Riskier Risk Seeking



Safer Risk Averse

Time

$$\delta^t_{r(A)}u(x)$$

Later Patient

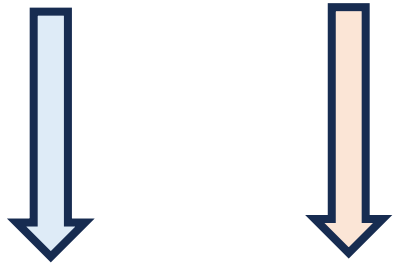


Sooner Impatient

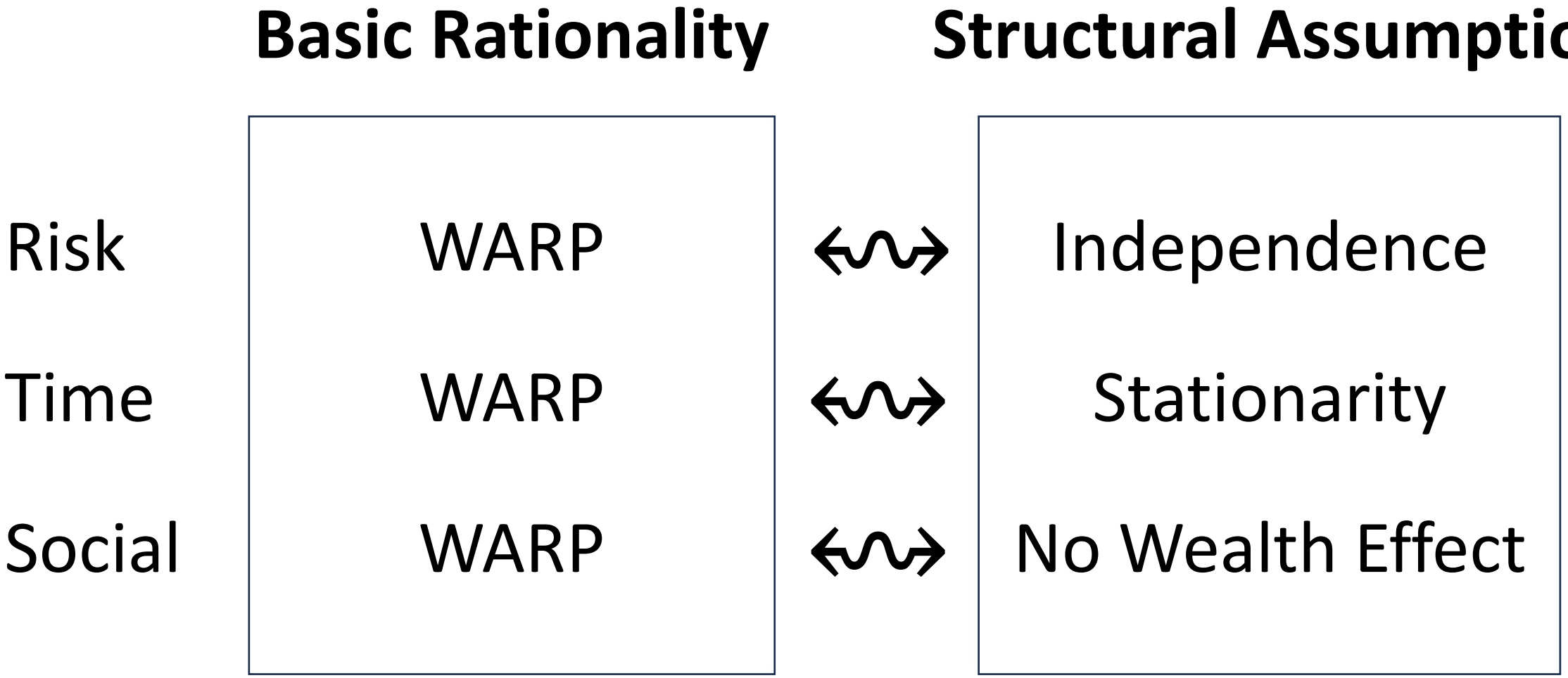
Social

$$x + u_{r(A)}(y)$$

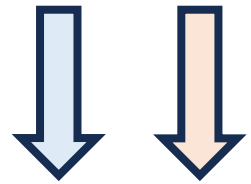
Disparity Selfish



Equality Altruistic


Basic Rationality
Structural Assumptions
Risk
Time
Social
WARP
WARP
WARP
Independence
Stationarity
No Wealth Effect

What we do

- **study**: changes in preferences “under” canonical functional forms
- **using**: ordered reference + ordered preference 
- **new view for**: structural violations in Risk, Time, Social
- **the key**: relate them to basic rationality violations

Risk

In-depth, then similarly Time and Social

Primitive

- $X \subset \mathbb{R}$ finite set of prizes
- $\Delta(X)$ all lotteries
- $\mathcal{A}$ all finite nonempty subsets of lotteries, $A \in \mathcal{A}$
- **Primitive**: $c: \mathcal{A} \to \mathcal{A}$ where $c(A) \subseteq A$

4 Axioms: R1-R4

- No weak order, WARP, acyclicity, etc.

Technical / basic:

- R1: Continuity

- R2: First Order Stochastic Dominance

R4

- **Definition**: Fix A, let $x \succcurlyeq_A y$ if $x \in c(A)$ and $y \in A$ $\quad$ $(\ldots \succ_A)$

Suppose $A \subset B$.

If $\alpha\delta \oplus (1-\alpha)q \succcurlyeq_A \alpha p \oplus (1-\alpha)q$

then not $\alpha\delta \oplus (1-\alpha)q' \prec_B \alpha p \oplus (1-\alpha)q'$

R4

- **Definition**: Fix A, let $x \succcurlyeq_A y$ if $x \in c(A)$ and $y \in A$ $\quad$ $(\ldots \succ_A)$

- R4:

Suppose $A \subset B$.

If $\quad \alpha\delta \oplus (1-\alpha)q \succcurlyeq_A \alpha p \oplus (1-\alpha)q$

then not $\quad \alpha\delta \oplus (1-\alpha)q' \prec_B \alpha p \oplus (1-\alpha)q'$

R3 (intuition)

$A = \{p, q, p', q'\}$ (where $p' = p^{\alpha}s$, $q' = q^{\alpha}s$)

$\{p, q, q'\}$ $\{p, q, p'\}$ $\{p, q'\}$ $\{q, q'\}$

$\{q, p', q'\}$ $\{q, p'\}$

$\{p, q, p', q'\}$

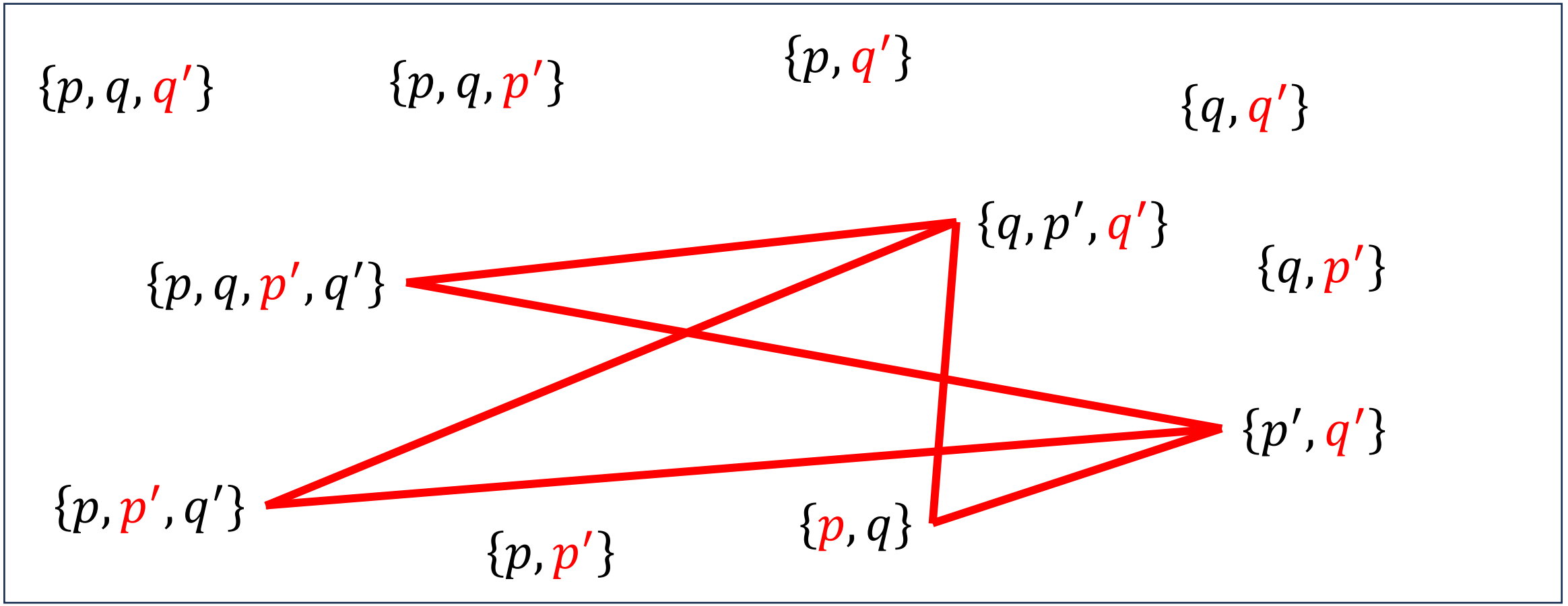
$\{p', q'\}$

$\{p, p', q'\}$ $\{p, q\}$

$\{p, p'\}$

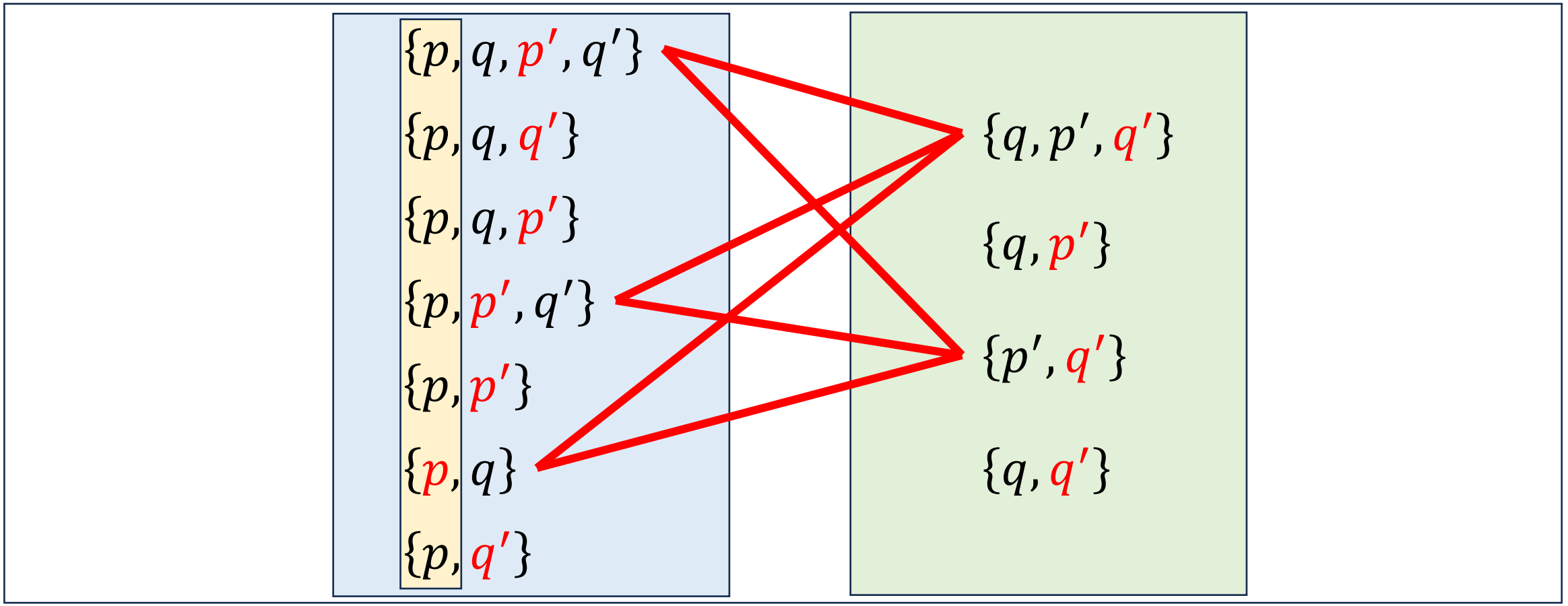
R3 (intuition)

$A = \{p, q, p', q'\}$ (where $p' = p^{\alpha}s$, $q' = q^{\alpha}s$)



R3 (intuition)

$A = \{p, q, p', q'\}$ (where $p' = p^{\alpha}s$, $q' = q^{\alpha}s$)



Local Independence

- **Definition**: c satisfies Independence (globally) if for all $A, B \in \mathcal{A}$ and common mixtures (p, q), (p', q'),

$$p \succcurlyeq q \Rightarrow \text{not } q' \succ p'$$

- **Definition**: c satisfies Independence over $\mathcal{S} \subseteq \mathcal{A}$ if for all $A, B \in \mathcal{S}$ and common mixtures (p, q), (p', q'),

$$p \succcurlyeq_A q \Rightarrow \text{not } q' \succ_B p'$$

R3: Risk Reference Dependence

- Let $\Phi(A) = A \backslash [MPS(A) \cup ES(A)]$
  - $p \in MPS(A)$ if $\exists q \in A$: p is a mean-preserving spread of q
  - $p \in ES(A)$ if $\exists q \in A$: p is q mixed with $\alpha\delta_b + (1-\alpha)\delta_w$

- R3:

For any $A \in \mathcal{A}$, $\exists p \in \Phi(A)$ such that
c satisfies Independence over $\{B \subseteq A: p \in B\}$.

R1-R4 ⇔ Ordered Expected Utility Model

There exists R on $\Delta(X)$ (reference order) and $\{u_r\}_{r\in\Delta(X)}$ such that

- R ranks *safer* alternatives higher
- u_r a concave transformation of $u_{r'}$ if rRr'

Riskier ⟹ Safer

Convex ⟹ Concave

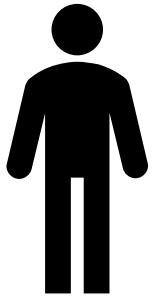
Step 1:

$$r(A) = \max(A, R)$$

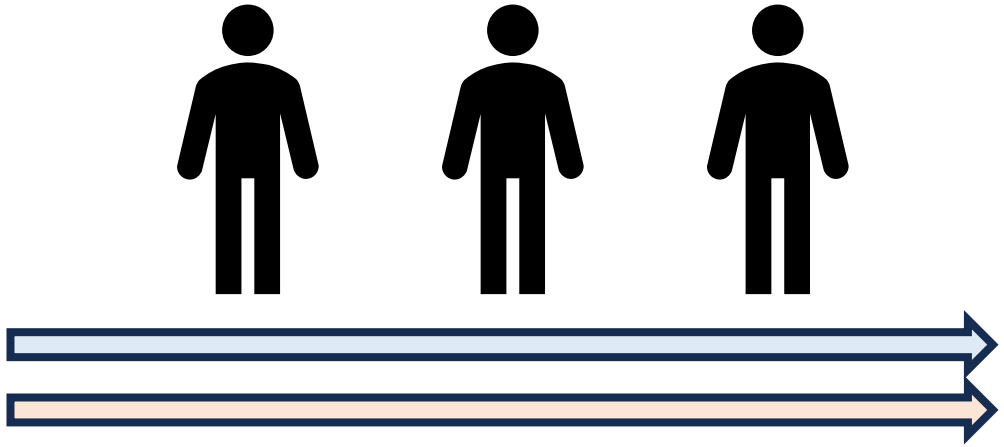
Step 2:

$$c(A) = \underset{p\in A}{\operatorname{argmax}} \sum p(x)u_{r(A)}(x)$$

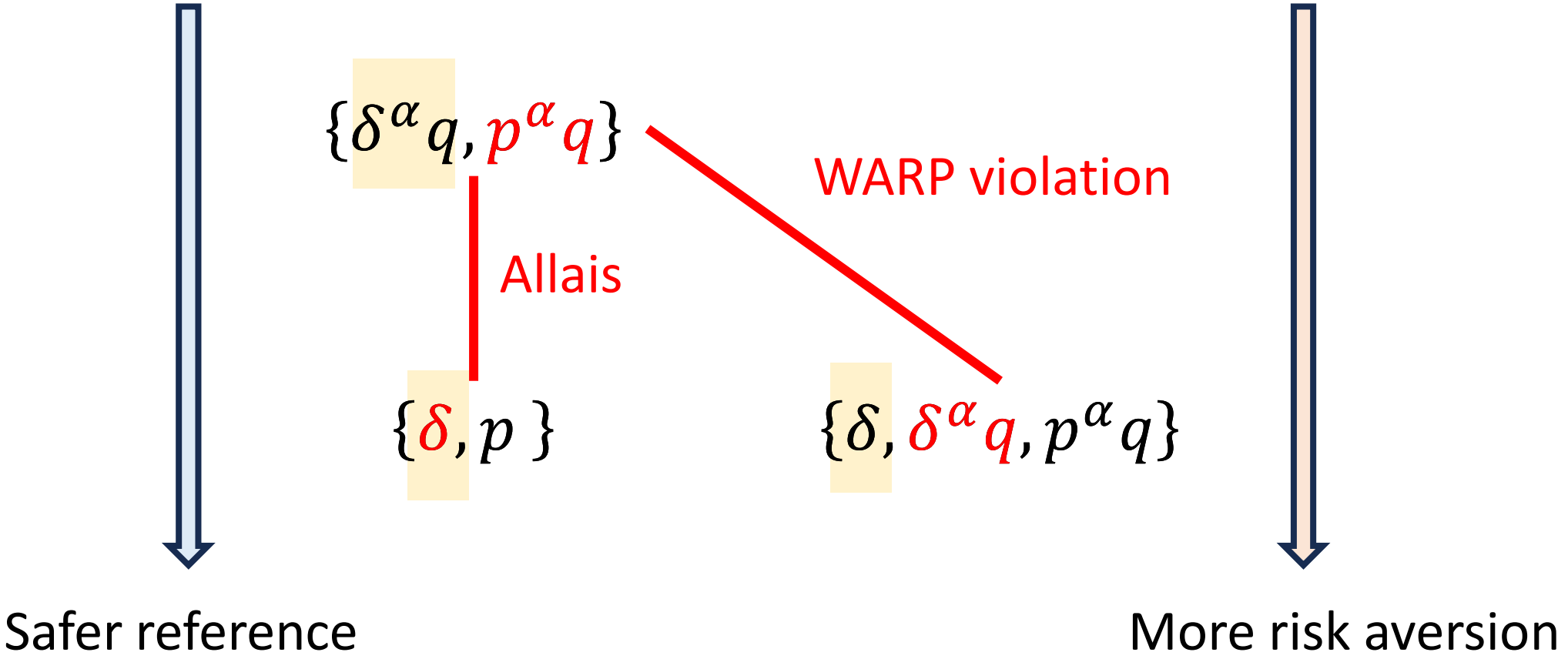
- **Locally**: standard EU



- **Globally**: ordered risk attitude



Independence violation ⇝ WARP violation



Identification

- R is identified given violations
- Fixed R, non-trivial u_r is unique up to positive affine transformation

Characterization

- **Q**: Is ordered reference important? **A**: Yes
- **Q**: Is ordered preference important? **A**: No, but important for later

Time

Alternatives: $X \times T$

Time Reference Dependence

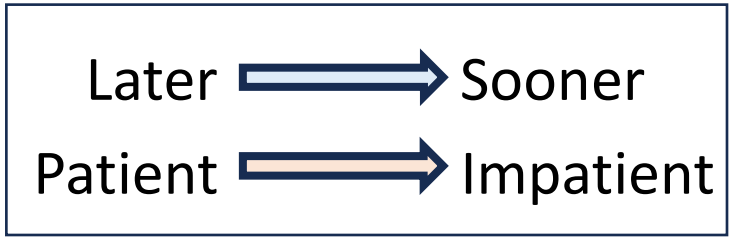
- Let $\Phi(A) = \{(x,t) \in A : t \leq t' \; \forall (x',t') \in A\}$

For any $A \in \mathcal{A}$, $\exists (x,t) \in \Phi(A)$ such that
c satisfies Stationarity over $\{B \subseteq A : (x,t) \in B\}$.

Axioms ⇔ Ordered Discounting Model

There exist u, R (reference order), $\{\delta_r\}_{r\in X\times T}$ such that

- R ranks sooner payments higher
- $\delta_r \leq \delta_{r'}$ if rRr'



Step 1:

$$r(A) = \max(A, R)$$

Step 2:

$$c(A) = \underset{(x,t)\in A}{\operatorname{argmax}}\, \delta_{r(A)}^{t} u(x)$$

Suppose $t < t', x < x'$

Single-switching

$\{(x, t+3), (x', t'+3)\}$

$\{(x, t+2), (x', t'+2)\}$

$\{(x, t+1), (x', t'+1)\}$

$\{(x, t+0), (x', t'+0)\}$

…Generalized

$\{(x, t+2), (x', t'+2)\}$

$\{(x, t+1), (x', t'+1)\}$

$\{(x, t+0), (x', t'+0), \dots\}$

OK NO OK

Social

Alternatives: $X \times Y$ (payment for me and for you)

Equality Reference Dependence

- Let $\Phi(A) = \min_{(x,y)\in A} \text{Gini}(x, y)$

For any $A \in \mathcal{A}$, $\exists (x, y) \in \Phi(A)$ such that
c satisfies No Wealth Effect over $\{B \subseteq A: (x, y) \in B\}$.

Axioms ⇔ Ordered Social Model

There exist R (reference order), $\{v_r\}_{r \in X \times Y}$ (other-regarding utility) …

- R ranks more equal distributions higher
- $v_r(y) - v_r(y') \geq v_{r'}(y) - v_{r'}(y')$ if $y > y'$ and rRr'

Step 1:

$$r(A) = \max(A, R)$$

Disparity ⟹ Equality

Selfish ⟹ Altruistic

Step 2:

$$c(A) = \underset{(x.y) \in A}{\operatorname{argmax}}\, x + v_{r(A)}(y)$$

Menu-dependent altruism

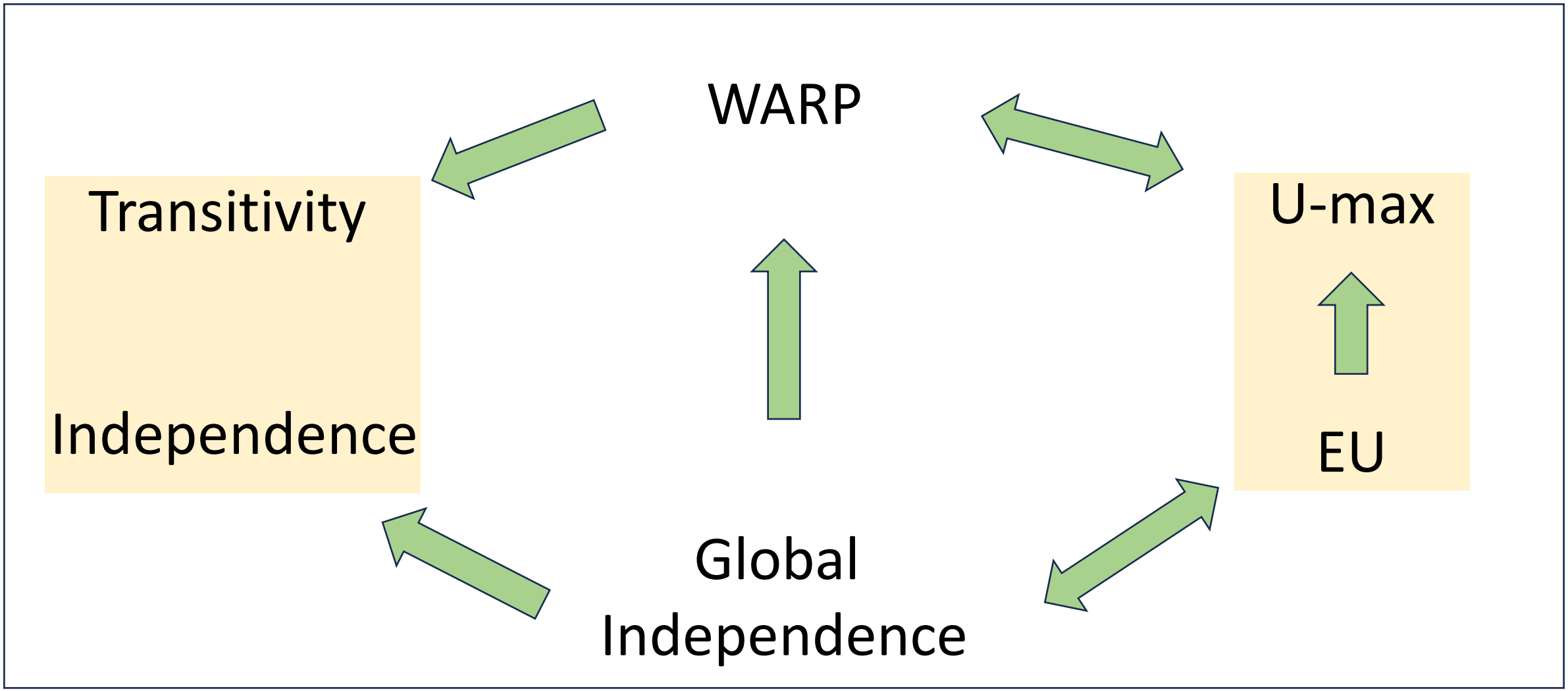
$\{(\$8, \$2), (\$7, \$3)\}$ $\{(\$8, \$2), (\$7, \$3), (\$5, \$5)\}$

Fairness vs efficiency

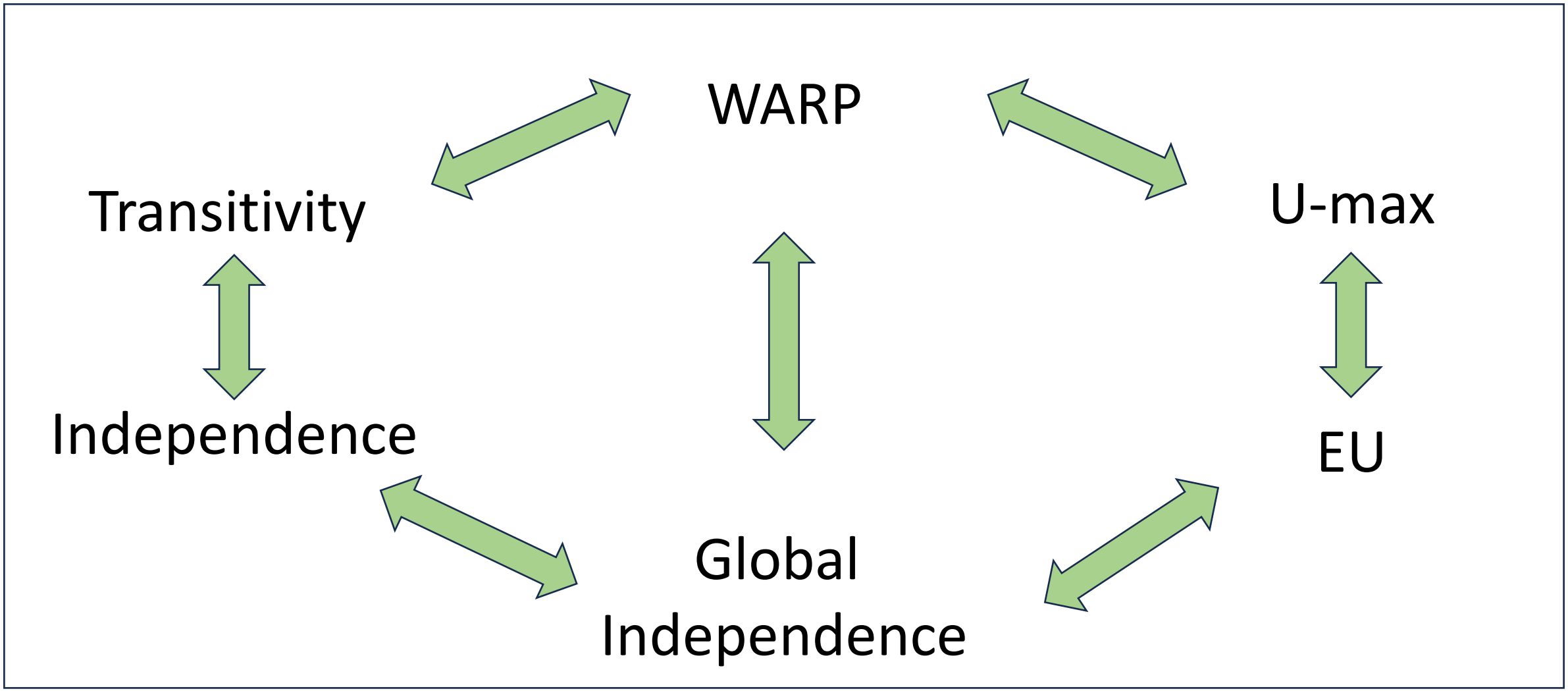
$\{(\$60, \$0), (\$30, \$20)\}$ $\{(\$60, \$0), (\$30, \$20), (\$25, \$25)\}$

Link

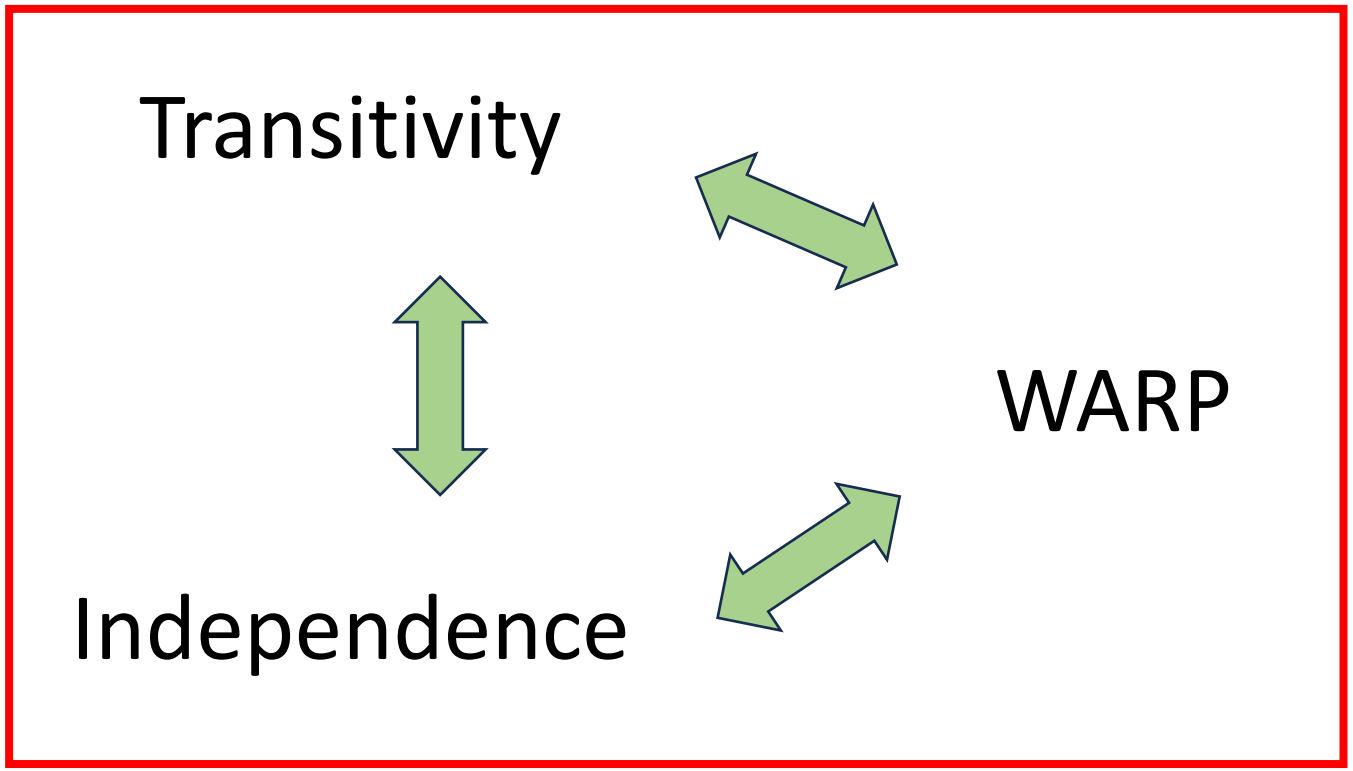
Lotteries, given continuous $c: \mathcal{A} \to \mathcal{A}$

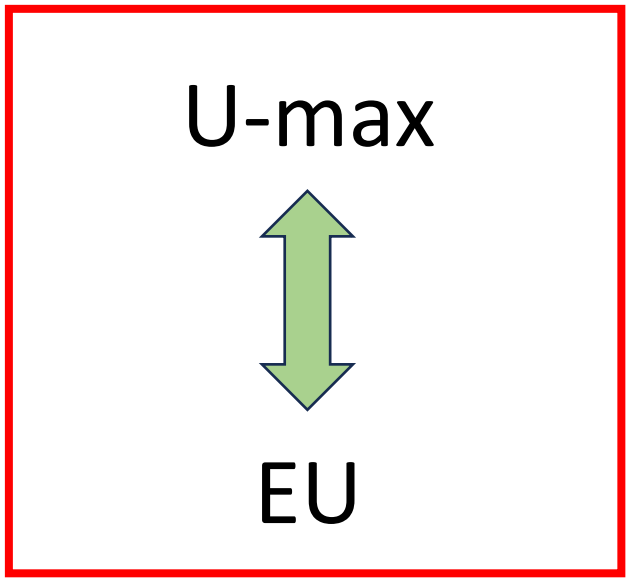


Ordered Expected Utility Model



Ordered Expected Utility Model





Ordered Risk Model

T ⇔ WARP ⇔ Independence ⇔ Expected Utility ⇔ U

Ordered Time Model

T ⇔ WARP ⇔ Stationarity ⇔ Exponential Discounting ⇔ U

Ordered Social Model

T ⇔ WARP ⇔ No Wealth Effect ⇔ Quasi-linear Utility ⇔ U

	Basic Rationality	Structural Assumptions
Risk	WARP	Independence
Time	WARP	Stationarity
Social	WARP	No Wealth Effect

Basic Rationality ⇜⇝ Structural Assumptions

…in a world of ordered preference

A Future

RUD EXCLUSIVE

Preference

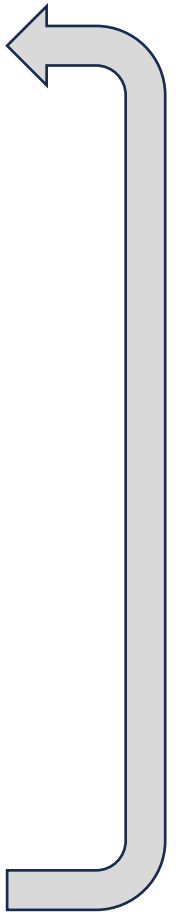
If $p \succcurlyeq q$, **either** require $p^{\alpha}s \succcurlyeq q^{\alpha}s$ **or** not.

Choice

Require $\{p^{\alpha}s, q^{\alpha}s, \dots\}$ only for **some** choice sets.

Choice with WARP

If $\{p^{\alpha}s, q^{\alpha}s, \dots\}$, then $\{p^{\alpha}s, q^{\alpha}s, \dots \dots\}$



Choice

Require $\{p^{\alpha}s, q^{\alpha}s, \ldots\}$ only for **some** choice sets.

Today: one of MANY possible examples:

For any $A \in \mathcal{A}$, $\exists z \in \Phi(A)$ such that
c satisfies $\mathcal{T}$ over $\{B \subseteq A : z \in B\}$.

Thank you

