An Analysis of Ordered Reference Dependent Utility

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1 Introduction

Lim (2023) characterized a series of models where failures of WARP and failures of structural postulates are inextricably linked. Although the focus is in richer choice domains (risk, time, and social preference), the starting point is the model of choice over generic alternatives called Ordered Reference Dependent Utility (Definition 4, Lim (2023)). The same model is also studied in other papers (Aumann and Serrano, 2008; Kıbrıs, Masatlioglu, and Suleymanov, 2023; Giarlotta, Petralia, and Watson, 2022), and in particular Kıbrıs, Masatlioglu, and Suleymanov (2023) provides a thorough comparison with several non-WARP models. This note potentially complements existing comparisons.

2 ORDU and other non-WARP models

To simplify notation, we use "$\{\underline{a}, b, c\}$" for $c(\{a, b, c\}) = \{a\}$.

Under comparable setups, ORDU neither nests nor is nested by any of the following models: (i) Ok, Ortoleva, and Riella (2015)'s *revealed (p)reference*, (ii) Kőszegi and Rabin (2006)'s *reference-dependent preferences* (*personal equilibrium*),

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(iii) Manzini and Mariotti (2007)'s *rational shortlist method*, and (iv) Masatlioglu, Nakajima, and Ozbay (2012)'s *choice with limited attention*.[1] The first two models involve reference formation. In the other two, the addition or removal of alternatives directly contribute to WARP violations.

A key observation separates ORDU from other models: In ORDU, because reference points are given by a reference order and choices maximize reference-dependent utilities, then either $c(\{a,b\})$ or $c(\{b,c\})$ must agree with $c(\{a,b,c\})$ in the maximization of a single utility function. This is because the reference point of $\{a,b,c\}$ is necessarily in $\{a,b\}$ or $\{b,c\}$ (or both). This defining feature of ORDU can be generalized:

Remark 1. Suppose c admits an ORDU representation and take any finite collection of choice problems $A_1,...,A_n$. For some $x \in A = A_1 \cup A_2 \cup ... \cup A_n$, choices between A and A_i such that $x \in A_i$ must comply with standard utility maximization.

In Ok, Ortoleva, and Riella (2015)'s model, the decision maker maximizes a single utility function but only chooses from alternatives that are better than the reference in all (endogenously determined) attributes. However, references do not necessarily come from an order. Consider the following choices accommodated by their model but not ORDU.[2]

$\{\underline{a},b,c,d\}$	$\{a,\underline{b},d\}$	$\{a,\underline{c},d\}$

In their model, a *decoy* d blocks the choice of a in $\{a,\underline{b},d\}$ and $\{a,\underline{c},d\}$ due to the *attraction effect,* where b and c are *elevated* because they are better than d in all attributes while a isn't. However, since reference formation is flexible as contrasted with the use of a reference order in ORDU, d need not be the decoy in $\{\underline{a},b,c,d\}$, resulting in the choice of a. On the contrary, ORDU requires that either the reference point of $\{a,b,d\}$ or the reference point of $\{a,c,d\}$ is the reference point of

[1] That is, for each of the four external models we consider, there exist choice correspondences that admit an ORDU representation but not the eternal model, and vice versa. Complete specifications of the choice correspondences are provided below.

[2] The complete choice correspondence is

$\{\underline{a},b,c,d\}$	$\{\underline{a},b,c\}$	$\{a,\underline{b},d\}$	$\{a,\underline{c},d\}$	$\{\underline{b},c,d\}$	$\{\underline{a},b\}$
$\{\underline{a},c\}$	$\{\underline{a},d\}$	$\{\underline{b},c\}$	$\{\underline{b},d\}$	$\{\underline{c},d\}$	

Using an Ok, Ortoleva, and Riella (2015) specification where $u(a) > u(b) > u(c) > u(d)$. $r(\{a,b,d\}) = r(\{a,c,d\}) = r(\{b,d\}) = r(\{c,d\}) = d$, $r(A) = \diamond$ otherwise, and $\mathcal{U} = \{U\}$ where $U(b) > U(c) > U(d) > U(a)$.

$\{a, b, c, d\}$. As a result, Remark 1 excludes this behavior from ORDU. On the other hand, intransitive behavior in binary choice problems, such as $\{\underline{a}, b\}, \{\underline{b}, c\}, \{\underline{c}, a\}$, can be explained by ORDU but are ruled out by Ok, Ortoleva, and Riella (2015), since the absence of a third alternative impedes their decoy-effect from taking place. Hence the two models are not nested.

Kőszegi and Rabin (2006)'s *reference-dependent preferences* is another related model. Gul and Pesendorfer (2006) provided the axiomatic foundation for *personal equilibrium* (PE), in which a decision maker has a joint utility function $v : X \times X \to \mathbb{R}$ and chooses $PE(A) = \{x : v(x|x) \geq v(y|x) \ \forall y \in A\}$. That is, the choice maximizes a reference-dependent utility function, and the reference point is itself the eventually chosen alternative (therefore "equilibrium"). This leads to the following behavior.[3]

$\{\underline{a}, b, c, d\}$	$\{\underline{a}, \underline{b}, c\}$	$\{\underline{a}, \underline{d}\}$

In this example, b is not chosen in $\{\underline{a}, b, c, d\}$. Yet it is chosen in the subset $\{\underline{a}, \underline{b}, c\}$, in which d—an alternative better than b under $v(\cdot|b)$—was removed. Furthermore, d is also chosen in the subset $\{\underline{a}, \underline{d}\}$ for the same reason—c, an alternative better than d under $v(\cdot|d)$—was removed. While ORDU also allows for $x \in c(B) \setminus c(A)$ where $B \subset A$, it does so with two implications: (i) an alternative $y \in A \setminus B$ must had been the reference point of A and therefore (ii) choices are consistent between $c(A)$ and $c(T)$ for all $T \subset A$ that contains y. This is not satisfied in our example: the first WARP violation implies that d is the reference point, yet the second WARP violation occurs while d is remains present. As a consequence, Remark 1 excludes this behavior from ORDU.

Conversely, an immediate implication of PE is, if $x \in c(A)$ and $x \in B \subset A$, then $x \in c(B)$. A simple intransitive choice pattern $\{\underline{a}, b\}, \{\underline{b}, c\}, \{\underline{c}, a\}$ is therefore admissible by ORDU but not PE.[4] We conclude that the two models are not nested.

[3] The complete choice correspondence is

$\{\underline{a}, b, c, d\}$	$\{\underline{a}, \underline{b}, c\}$	$\{\underline{a}, b, \underline{d}\}$	$\{\underline{a}, c, d\}$	$\{b, \underline{c}, d\}$	$\{\underline{a}, \underline{b}\}$
$\{\underline{a}, c\}$	$\{\underline{a}, \underline{d}\}$	$\{\underline{b}, \underline{c}\}$	$\{b, \underline{d}\}$	$\{\underline{c}, d\}$	

Gul and Pesendorfer (2006) shows that PE is equivalent to choices maximizing a complete (but not necessarily transitive) preference relation. This choice correspondence is explained by $a \sim b$, $a \succ c$ $a \sim d$, $b \sim c$, $d \succ b$, $c \succ d$.

[4] Gul and Pesendorfer (2006) shows that Kőszegi and Rabin (2006)'s *personal equilibrium* is equivalent to the maximization of a complete (but not necessarily transitive) preference relation.

Manzini and Mariotti (2007) proposes a non-WARP model without a reference point interpretation. In *rational shortlist method* (RSM), decision makers are endowed with two asymmetric relations P_1 and P_2. Facing a choice problem A, she first creates a shortlist by eliminating inferior alternatives according to P_1 (eliminate x if yP_1x for some $y \in A$), and then chooses from this shortlist according to P_2. WARP violation occurs when an alternative x is eliminated in a set S but not in its subset $T \subset S$, where x is subsequently chosen over the choice from S. An example of this behavior is the follow choice pattern.[5]

$$\boxed{\{\underline{a}, b, c, d\} \quad \{a, b, \underline{d}\} \quad \{b, \underline{c}, d\} \quad \{\underline{b}, c\}}$$

For ORDU to reconcile with this behavior, c must be deemed the reference of $\{a, b, c, d\}$, but then choices from $\{b, c, d\}$ and $\{b, c\}$ must comply with standard utility maximization. This behavior is therefore excluded by Remark 1.

RSM, however, cannot accommodate an alternative that makes it to the shortlists for a small and large choice problems (A_1 and A_2 where $A_1 \subset A_2$) but not an intermediate one (B where $A_1 \subset B \subset A_2$), generating choices accommodated by ORDU but not RSM, such as the following.[6]

$$\boxed{\{\underline{a}, b, c, d\} \quad \{a, \underline{b}, d\} \quad \{\underline{a}, b\}}$$

The intuition is that while ORDU is constrained when reference points are fixed, the model is more flexible than RSM when reference points change, since no restriction is put on the new utility functions.

Since this is not the case, ORDU does not nest RSM. While ORDU is constrained by fixed reference points, the model is more flexible than RSM when reference points do change, since no restriction is put on the new utility function. RSM, however,

[5]The complete choice correspondence is

$$\boxed{\begin{array}{cccccc} \{\underline{a}, b, c, d\} & \{\underline{a}, b, c\} & \{b, \underline{c}, d\} & \{\underline{a}, c, d\} & \{a, b, \underline{d}\} & \{\underline{a}, b\} \\ \{\underline{a}, c\} & \{a, \underline{d}\} & \{\underline{b}, c\} & \{b, \underline{d}\} & \{\underline{c}, d\} & \end{array}},$$

induced by (aP_1b, aP_1c, cP_1d, dP_1b) and (aP_2b, aP_2c, dP_2a, bP_2c, dP_2b, cP_2d).

[6]The complete choice correspondence is

$$\boxed{\begin{array}{cccccc} \{\underline{a}, b, c, d\} & \{a, \underline{b}, c\} & \{a, \underline{b}, d\} & \{\underline{a}, c, d\} & \{\underline{b}, c, d\} & \{\underline{a}, b\} \\ \{\underline{a}, c\} & \{\underline{a}, d\} & \{\underline{b}, c\} & \{\underline{b}, d\} & \{\underline{c}, d\} & \end{array}}.$$

This is explained by the ORDU specification: $bRaRcRd$, $u_i(a) > u_i(b) > u_i(c) > u_i(d)$ when $i \in \{a, b, d\}$, and $u_c(b) > u_c(a) > u_c(c) > u_c(d)$.

cannot accommodate a choice that makes the shortlists in a small and large set but not an intermediate one. We conclude that ORDU and RSM are not nested.

Last, we compare ORDU to Masatlioglu, Nakajima, and Ozbay (2012)'s *choice with limited attention* (CLA). A decision maker has a complete and transitive ranking $\succ_{CLA}$ of alternatives and an attention filter that limits choices to a subset of each choice problem, the "consideration set". When another choice problem is derived by removing choices not originally considered, the consideration set remains the same. Although a single ranking is used—as opposed to ORDU's many utility functions—flexibility in constructing consideration sets easily allows for behavior not accommodated by ORDU, for example the following, which is excluded from ORDU by Remark 1.

$\boldsymbol{\{a, \underline{b}, c\}}$	$\boldsymbol{\{\underline{a}, b\}}$	$\boldsymbol{\{b, \underline{c}\}}$	$\{\underline{a}, c\}$

Interestingly, CLA is provided under the framework of choice functions (no indifference), and with that restriction ORDU is nested by CLA.[7] However, the two models make different predictions when indifferences are allowed. For the analysis, we modify CLA by allowing for indifferences in the ranking of alternatives (replacing $\succ_{CLA}$ with $\succsim_{CLA}$), but preserve in entirety the attention filter / consideration set component. The following behavior is accommodated by ORDU but not CLA.[8]

$\boldsymbol{\{\underline{a}, \underline{b}, c, d\}}$	$\boldsymbol{\{a, \underline{b}, \underline{c}\}}$	$\boldsymbol{\{\underline{b}, c\}}$

When indifferences are allowed, the single ranking limitation of CLA becomes the bottleneck in explaining behavior. The two models are hence not nested under comparable setups.

[7] Consider any choice function c that admits an ORDU representation, define CLA's parameters as follows: attention filter $\Gamma(A) := \left\{\min(A, R), \arg\max_{x \in A} u_{\min(A,R)}(x)\right\}$ (singleton if $\min(A, R) = \arg\max_{x \in A} u_{\min(A,R)}(x)$) and CLA's preference $x \succ y$ if xRy.

[8] The complete choice correspondence is

$\boldsymbol{\{\underline{a}, \underline{b}, c, d\}}$	$\boldsymbol{\{a, \underline{b}, \underline{c}\}}$	$\{\underline{a}, \underline{b}, d\}$	$\{\underline{a}, c, d\}$	$\{\underline{b}, c, d\}$	$\{a, \underline{b}\}$
$\{a, \underline{c}\}$	$\{\underline{a}, d\}$	$\boldsymbol{\{\underline{b}, c\}}$	$\{\underline{b}, d\}$	$\{\underline{c}, d\}$	

This is explained by the ORDU specification: $cRbRaRd$, $u_d(a) = u_d(b) > u_d(c) > u_d(d)$, $u_a(b) = u_a(c) > u_a(a)$, $\underline{u_b(b) > u_b(c)}$. Now we show non-compliance with CLA (with the indifference extension): Since $a \in c(\{a, b, c, d\}) \setminus c(\{a, b, c\})$, CLA reconciles this by setting the consideration sets $\Gamma(\{a, b, c, d\}) = \{a, b, d\}$ and $\Gamma(\{a, b, c\}) = \{b, c\}$, so a is not considered in the smaller set. However, the property of consideration sets then requires $\Gamma(\{b, c\}) = \{b, c\}$, and $\{a, c\}$

References

AUMANN, R. J., AND R. SERRANO (2008): "An Economic Index of Riskiness," *Journal of Political Economy*, 116(5), 810–836.

GIARLOTTA, A., A. PETRALIA, AND S. WATSON (2022): "Semantics Meets Attractiveness: Choice by Salience," *arXiv preprint arXiv:2204.08798*.

GUL, F., AND W. PESENDORFER (2006): "The Revealed Preference Implications of Reference Dependent Preferences," *manuscript, Princeton University*.

KIBRIS, Ö., Y. MASATLIOGLU, AND E. SULEYMANOV (2023): "A Theory of Reference Point Formation," *Economic Theory*, 75, 137–166.

KŐSZEGI, B., AND M. RABIN (2006): "A Model of Reference-Dependent Preferences," *Quarterly Journal of Economics*, 121(4), 1133–1165.

LIM, X. Z. (2023): "Ordered Reference Dependent Choice," *arXiv preprint arXiv:2105.12915*.

MANZINI, P., AND M. MARIOTTI (2007): "Sequentially Rationalizable Choice," *American Economic Review*, 97(5), 1824–1839.

MASATLIOGLU, Y., D. NAKAJIMA, AND E. Y. OZBAY (2012): "Revealed Attention," *American Economic Review*, 102(5), 2183–2205.

OK, E. A., P. ORTOLEVA, AND G. RIELLA (2015): "Revealed (P)Reference Theory," *American Economic Review*, 105(1), 299–321.