Informative Consumption

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This version: 2026/06/20
Most recent public version: http://s.xzlim.com/ic

Abstract

Risky consumption generates information when uncertainty is resolved. This paper axiomatically characterizes the consumption–information trade-off even when the analyst does not observe an agent's future problems. A subjective future menu underpins the agent's willingness to sacrifice current consumption for future information. By carefully separating objective risk from subjective risk, we decompose the certainty equivalent of an act into a standard risk premium and a novel information premium. To facilitate applications, we introduce an Arrow–Debreu–Pratt parameterization that yields a tractable model, capturing risk aversion and information incentives with a single coefficient for each. Finally, we show that heterogeneity in risk-taking may arise from differing opportunities to capitalize on information, rather than being solely attributable to differences in risk aversion.

Keywords: Subjective expected utility, experimentation, Blackwell order, information premium
JEL: D01, D11

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1 Introduction

Economic agents often have to choose an action not knowing for certain what the outcome will be. Risky consumption decisions inherently contain information when uncertainty is resolved, rewarding the agent not only in terms of today's utility (consumption incentive) but also through better future decisions (information incentive). Disentangling the two incentives helps address why population heterogeneity in risk-taking behavior is not solely the result of heterogeneous risk attitudes, but also the result of heterogeneous future opportunities, where some individuals can benefit more from (the same) information than other individuals do.

Information incentives are ubiquitous, even if the analyst does not observe it directly. Consider the well-known *Anscombe–Aumann act*, the predominant tool to study choice under (subjective) risk. It prescribes the probability of different outcomes $x, y, z, ...$ under each state of the world $\omega_1, \omega_2, ...$, general enough to find applications across all of economics. For example, the act f is risky but g always delivers the same outcome z.

f	x	y
ω_1	0.9	0.1
ω_2	0.3	0.7

g	z
ω_1	1
ω_2	1

When the DM does not know the true state ω, she forms a belief about the likelihood of different states and outcomes, and uses that belief to make consumption decisions. However, upon choosing f, outcome x is substantially more likely to occur under state ω_1 than under state ω_2, so when uncertainty is resolved and x occurs, a Bayesian DM receives new information suggesting that the state is quite likely ω_1. This information may have value depending on what the DM's future problems entail, even if they fall outside the scope of the analyst's observations.

In fact, an Anscombe–Aumann act is structurally identical to a *Blackwell experiment*, and f Blackwell-dominates the uninformative g. The DM may hence have an incentive to choose f even when its consumption utility is lower than that of g, and analogously an incentive to choose g even though it is Blackwell-dominated by f.

This simple observation is consistent with a wide range of real-life problems. A diner trying a new restaurant learns about its quality while enjoying the meal. A doctor administering a treatment observes the patient's clinical response and learns

about the treatment's effectiveness. A firm deploying a marketing strategy will learn about features of consumer demand.

The extent to which (the same) information has value often depends on unobserved individual characteristics: whether the diner will soon move out of town, whether the doctor is aware of new future treatments, whether the firm has a certain confidential expansion plan.

To disentangle this unobserved consumption–information trade-off, we axiomatically characterize a model called **Informative Consumption (IC)**. The primitive is simply preference over Anscombe–Aumann acts. In the model, the DM has a *subjective future menu*, A, which is ultimately inferred from choice behavior. Along with other usual preference parameters, the DM chooses act f to maximize

$$\alpha U_C(f) + (1-\alpha) U_I(f), \tag{1.1}$$

where $U_C(f)$ is the standard, first-period expected consumption utility, $U_I(f)$ is a continuation value from information acquisition, and α is a consumption–information weight. The continuation value $U_I(f)$ depends on both the information content in f and the options in A. The model is formally given in Section 2.

One application for the weight is discounting, where the DM enjoys an immediate consumption utility and a discounted future utility. She therefore maximizes

$$U_C(f) + \beta U_I(f)$$

where $\beta = (1-\alpha)/\alpha \in [0,1]$.

IC admits the standard (one-period) Subjective Expected Utility (SEU) as a special case when $\alpha = 1$ (or $\beta = 0$). But unlike essentially all generalizations of SEU (where the focus has predominantly been on the violations of expected utility forms or the presence of multiple priors), IC departs from SEU not due to behavioral biases. Quite the contrary, the DM is sophisticated, forward-looking, and takes information incentives into account.

In fact, the main axiom *Mixture Aversion* is the exact opposite of the well-known maxmin expected utility's axiom Uncertainty Aversion (Gilboa and Schmeidler, 1989). When the DM is indifferent $f \sim g$, mixing the acts weakens the information content of signal realizations because they are now more equally distributed across

states. Hence $f \succ af + (1-a)\, g$ when the DM is not indifferent to the mixture.

Risk–Information Decomposition

IC enables the study of a novel notion called *information premium* that is in many ways analogous to the well-known *risk premium*. In this framework, both premiums play a role in determining a DM's certainty equivalent of an act. Because a certainty equivalent is certain, the (degenerate) act that offers the certainty equivalent is uninformative, so the DM must be compensated for the loss of information value as she gives up an act. We show that information premium has intuitive properties. The information premium of an uninformative act f is zero. Under an intuitive parameterization, proposition 4 further shows that Blackwell-dominating acts have greater information premiums. It also shows that information premiums weakly increase for all acts when information incentive is greater, which is intuitive because there is more valuable use of the same information. The difference between an act's expected payoff and its certainty equivalent is exactly the sum of risk premium and information premium, both of which can be identified by carefully separating objective risk (risky but no information) from subjective risk (contains information).

Behavioral Characterization

The characterization of IC involves four key axioms (and a technical condition). Mixture Aversion is already mentioned. Skewness Independence relaxes the standard independence condition by applying it only to mixtures that involve proportional likelihoods in generating the same prize across different states. Intuitively, the new axiom allows for a DM to care about whether a prize z occurs through state ω_1 or state ω_2, for instance, which is not the case in standard SEU where the DM simply calculates the aggregate probability of z. Of course, mixing proportional acts does not alter information content because posteriors are determined by likelihood ratios. The last two significant axioms are Prize Independence and State-Prize Independence. The former weakens monotonicity and says that an "upgraded" act should be preferred if the upgrade does not involve overlapping prizes; again, the intuition is about not distorting the likelihood of a prize across different states. The latter requires that equal and state-dependent distortions do not change preferences.

Arrow–Debreu–Pratt Parameterization

For applications, Section 4 introduces a 2-parameter parameterization of IC where ρ captures risk aversion and ι captures information incentive. It is common to work with CARA functional forms where the Arrow–Pratt coefficient ρ captures the degree of risk aversion. It turns out that the future menu A can also have an intuitive parameterization using a menu of Arrow–Debreu assets, each paying the prize ι in a particular state. An individual with a greater ι has stronger information incentive and always enjoys more information premiums. We show that this parameterization greatly simplifies IC. In particular, continuation value is now separable between preference parameters (ρ, ι) and a novel information index $\mathbb{I}(f, \mu)$ that captures the aggregate information value of f under prior μ (proposition 3). Results show that differences in risk-taking behavior can be reconciled using the same degree of risk aversion but with the more risk-taking DM having a greater information coefficient ι if her chosen act has a greater information index (proposition 5).

Section 2 introduces the primitive, the model, and the risk–information decomposition. Section 3 proposes axioms and state the representation theorem. Section 4 proposes the Arrow–Debreu–Pratt representation. Section 5 concludes. Appendix A contains proofs.

1.1 Related Literature

In the subjective learning literature that can be traced back to Kreps (1979), theories use observed preferences over menus to infer the possibility that a DM expects to receive information tomorrow, even if that information is not observed by the analyst. Most work focuses on **exogenous subjective learning**, where the DM has no control over what she might learn (Dekel et al., 2001; Dillenberger et al., 2014). Then comes **endogenous subjective learning**, in which the DM has some control (Hyogo, 2007; Piermont et al., 2016; Cooke, 2017). In this literature, learning is subjective because the analyst does not observe the information structures of the DMs.

We study a different problem where information content is known because Anscombe–Aumann acts are structurally identical to Blackwell experiments. Instead, our main unobservable is the (future) menu that the DM expects to face,

which is exogeneously observed in most of the subjective learning literature.

Like us, Liang (2017) and Rommeswinkel et al. (2023) study the **consumption–information trade-off with known information content**. Our work was developed independently, though all three papers analyze closely related models. Liang (2017) introduces a general framework of *information-dependent expected utility* and considers a close variant of IC as special case,[1] whereas we highlight the role of the subjective future menu and develop analytical tools and tractable parameterizations. The two papers also propose different behavioral foundations (axioms) with theirs emphasize an information-based perspective whereas our main axiom, Mixture Aversion, captures the central tension of the consumption–information trade-off. Rommeswinkel et al. (2023), differently, study Savage acts, so learning occurs only at the event level, and they identify the informational value of partitions of the state space, which serve as a general foundation for the *preference for knowledge*.

There is also rising interest in the **intrinsic preference for information**, where a DM has preference over the resolution of uncertainty even if that information cannot help future decisions (Masatlioglu et al., 2023; Gul et al., 2025; Ke and Zhang, 2025). In contrast, we study what may be more accurately described as extrinsic preference for information because they can improve future decisions. This point is also made by Ergin and Sarver (2015), who also introduce *hidden actions*, which are related to our subjective future menu, although the functions of hidden actions / future menus are different. In theory, decision making can be affected by both forms of preference for information. Our work shows that distinguishing objective risk from subjective risk can separate the two even if extrinsic information incentives are not directly observed.

There is also a literature on **pure information acquisition** without consumption, in which we are most related to two sub-strands. One sub-strand characterizes **costless information acquisition** (Gilboa and Lehrer, 1991; Azrieli and Lehrer, 2008; Frankel and Kamenica, 2019). We share technical methods because the need to infer a DM's unobserved problem is in common. Our study disentangles information and consumption incentives in the presence of unobserved informa-

[1]Excluding technical differences regarding continuity, their *hidden acts* representation, which is related to Azrieli and Lehrer (2008), captures hidden information incentive using a menu of utility vectors.

tion–consumption trade-off. The second sub-strand takes **information cost** into the picture. Caplin and Dean (2015) characterize optimal information acquisition, De Oliveira et al. (2017) identify information acquisition cost, and Caplin et al. (2022) characterize Shannon cost. Unlike our study, decision problems are observed. We note that our approach may contribute to microfounding information cost using consumption loss.

2 Model

Let the set of prizes be $X = \mathbb{R}$, and let $\Omega = \{\omega_1, ..., \omega_N\}$ be a finite state space with $|\Omega| = N$. Let $\Delta(X)$ be the space of simple lotteries, i.e., lotteries with finite support. An (Anscombe–Aumann) act is a function $f : \Omega \to \Delta(X)$ and $\mathcal{F}$ is the set of all acts. The primitive is a binary relation $\succsim$ over $\mathcal{F}$. The symmetric and asymmetric components of $\succsim$ are denoted by $\sim$ and $\succ$, respectively.

Notation: Per convention, $p(x)$ denotes the probability that lottery p gives prize x and $f(x \mid \omega)$ denotes the probability that act f gives prize x under state ω. A *degenerate lottery* δ_x gives prize x with probability 1. A *constant act* $p \in \mathcal{F}$ gives lottery $p \in \Delta(X)$ in every state. A *degenerate act* $\delta_x \in \mathcal{F}$ is a constant act that gives the degenerate lottery δ_x. For $f, g \in \mathcal{F}$ and $a \in [0,1]$, the *mixture act* $af + (1-a)g$ denotes the act that gives prize x under state ω with probability $af(x \mid \omega) + (1-a)g(x \mid \omega)$. As is standard, $\text{supp}(f) \coloneqq \{x \in X : \sum_\omega f(x \mid \omega) > 0\}$ for all $f \in \mathcal{F}$ and $\text{supp}(p) \coloneqq \{x \in X : p(x) > 0\}$ for all $p \in \Delta(X)$. For notational brevity, we use $f \cap g$ to refer to $\text{supp}(f) \cap \text{supp}(g)$. For later, given $A \subseteq \mathcal{F}$, let $\text{supp}(A) := \cup_{f \in A} \text{supp}(f)$. We use the convention that $0 \times -\infty$ is 0.

2.1 Informative Consumption Representation

Even though our observation is preference over (one period's) acts, the DM in our model is, as if, facing a two-period problem. Given a prior over states $\mu \in \Delta(\Omega)$ and a Bernoulli utility function $u : X \to \mathbb{R}$, her choice of act f gives her the standard (subjective) expected utility in the **first period**,

$$U_C(f) = \sum_{\omega \in \Omega} \mu(\omega) \sum_{x \in \text{supp}(f)} f(x \mid \omega)\, u(x). \tag{2.1}$$

In the **second period**, the DM solves an (unobserved) problem utilizing the information obtained from the resolution of f, which gives her an (ex-ante) continuation value

$$U_I(f) \coloneqq \sum_{x \in \text{supp}(f)} \underbrace{\left[\sum_{\omega \in \Omega} \mu(\omega) f(x \mid \omega)\right]}_{\text{probability of signal/prize } x} \underbrace{\left[\max_{g \in A} \sum_{\omega \in \Omega} \mu_{x,f}(\omega) \sum_{y \in \text{supp}(g)} g(y \mid \omega) u(y)\right]}_{\text{value of posterior } \mu_{x,f}}. \tag{2.2}$$

The formulation of continuation value is standard: Upon receiving a prize x, the DM forms a Bayesian posterior $\mu_{x,f} \in \Delta(\Omega)$, i.e.,

$$\mu_{x,f}(\omega) = \frac{\mu(\omega) f(x \mid \omega)}{\sum_{\omega' \in \Omega} \mu(\omega') f(x \mid \omega')} \forall \omega.$$

Then, she chooses from her perceived future menu A to maximize expected utility, which results in the value of having posterior $\mu_{x,f}$. Finally, by aggregating over possible prize realizations $x \in \text{supp}(f)$, she obtains her continuation value.

Definition 1. $\succsim$ admits an **Informative Consumption (IC)** representation if there exist a full support prior $\mu \in \Delta(\Omega)$, a strictly increasing utility function $u : X \to \mathbb{R}$, a consumption-information weight $\alpha > 0$, and a future menu $A \subseteq \mathcal{F}$ such that $\succsim$ is represented by the function $U : \mathcal{F} \to \mathbb{R}$ where

$$U(f) = \alpha U_C(f) + (1 - \alpha) U_I(f).$$

The model departs from standard SEU, but it is not due to behavioral biases. Instead, the DM is more sophisticated in the sense that she considers not only today's consumption but the information benefits on future decisions. Note that $A \subset \mathcal{F}$ is a *subjective future menu*. It is neither decided nor known by the analyst, as it is not part of the primitive. Instead, it captures the DM's perception of what the future entails. It could be correct but falls outside of the analyst's observation, or it could simply be the DM's misconception. In any case, this perceived future menu captures an extrinsic preference for information that is hidden to the analyst.

The special case where $\alpha = 1$ corresponds to a DM who only cares about consumption value, and we obtain standard SEU. But even if $\alpha < 1$, information incentive can still be absent when A is a singleton, that is, the DM does not have

a choice to make in the future. In this case, information may be present but it has no value, and in particular $U_I(f) =$ constant for all f. Clearly, maximizing $U(f) = U_C(f) +$ constant is standard SEU. Section 3 characterizes IC and related representations. Section 4 parameterizes A with a (single) coefficient of information incentive.

2.2 Risk–Information Decomposition

Risk premium has served as an incredibly useful measure to summarily capture an economic agent's risk aversion. We propose a novel notion of *information premium* because risk-taking behavior can also be affected by information incentives.

The *certainty equivalent* of f, denoted by $CE(f)$, is a prize in X such that receiving that sure prize (in every state) is as good as receiving f, i.e., $\delta_{CE(f)} \sim f$. For a given distribution of prizes, standard methods infer that a lower certainty equivalent is associated with greater risk aversion.

This is no longer true in our model. The certainty equivalent can still be well-defined, but it now accounts for two distinct forces. The first is standard. Because certainty is preferred by a risk averse DM, a certainty equivalent tends to be lower than the mean of an act where the difference is usually called the *risk premium*. This is the only force for a constant act $p \in \mathcal{F}$ (it gives the lottery $p \in \Delta(X)$ in every state of the world), so we obtain the standard

$$\mathbb{E}_p(x) - CE(p) = \text{risk premium}.$$

The second force is novel. Because the degenerate act $\delta_{CE(f)}$ is uninformative, the certainty equivalent of f must use pure consumption to compensate for forgone information utility from f, so it leads to a greater certainty equivalent due to what we call *information premium*.

A typical act $f \in \mathcal{F}$ involves both premiums and we propose a decomposition. Let p_f be the *lottery reduction* of f, i.e., $p_f(x) = \sum_\omega \mu(\omega) f(x \mid \omega)$. Consider

$$\mathbb{E}_f(x) - CE(f) = \underbrace{[\mathbb{E}_f(x) - CE(p_f)]}_{\text{RP}(f)\text{ (risk premium)}} + \underbrace{[CE(p_f) - CE(f)]}_{-\text{IP}(f)\text{ (information premium)}} \tag{2.3}$$

where $\mathbb{E}_f(x) = \sum_{x \in \text{supp}(f)} \sum_\omega \mu(\omega) f(x \mid \omega) x$.

Rearranging Equation 2.3 gives

$$CE\left(f\right)=\mathbb{E}_{f}\left(x\right)-\text{RP}\left(f\right)+\text{IP}\left(f\right). \tag{2.4}$$

Information premium IP compensates the DM for giving up information. Therefore, it summarily captures the influence of a DM's information incentive on her evaluation of risky prospects. It is non-negative. If f is a constant act, then f is identical to its reduction p_f and $\text{IP}\left(f\right)=0$. This is intuitive since a constant act does not provide information. We show in proposition 4, under a parameterization, that information premium is analogous to the familiar risk premium in the way it reacts to preference parameters and the nature of different acts.

2.3 Information and Subjective Risk

The decomposition into different premiums captures how *objective risk* and *subjective risk* differently affect the evaluation of an act.

Objective risk refers to the uncertainty from an objective distribution of prizes, such as the lottery given in a particular state $f\left(\omega\right)$. Differently, subjective risk refers to the uncertainty due to the DM's subjective prior about states interacted with state-dependent outcomes. For instance, act f has subjective risk but no objective risk, and act g has objective risk but no subjective risk.

f	x_1	x_2
ω_1	1	0
ω_2	0	1

g	x_1	x_2
ω_1	$1/4$	$3/4$
ω_2	$1/4$	$3/4$

Standard models in economics, such as the SEU, evaluate both forms of risks using expected utility. In the example above, if $\mu\left(\omega_1\right)=1/4$, then the lottery reductions p_f, p_g are identical and SEU deems f, g indifferent. The presence of information incentive causes a departure from SEU, but it only does so through subjective risk.

In the decomposition (Equation 2.3), because p_f is a constant act containing only objective risk, which means it is structurally identical to an uninformative Blackwell experiment, risk premium is not affected by subjective risk after controlling for aggregate risk (the marginals of each prize).

On the other hand, p_f and f have the same aggregate probabilities of prizes, so they should lead to the same consumption utilities. Their differences come from the subjective risk contained in f (interacting with μ), which provides information when resolved. Therefore, information premium is the result of the interaction between information incentive and subjective risk.

If A is a singleton (the DM thinks that she has no future choice to make), we return to standard SEU. But even if A is meaningfully non-trivial, constant acts can never contain information because they do not contain subjective risk, their information premiums are still 0.

Section 4 continues this discussion under an intuitive parameterization.

Information-induced risk

We remark that there is a necessary trade-off between information and risk. The reduction p_f of f is degenerate if and only if there exists $z \in X$ such that $f(z \mid \omega) = 1$ for all $\omega \in \text{supp}\mu$. In that case, $\text{IP}(f) = 0$ and $\text{RP}(f) = 0$.

If $\text{IP}(f) > 0$, then p_f is non-degenerate. If u is strictly risk averse (i.e., strictly concave), then $\text{IP}(f) > 0$ implies $\text{RP}(f) > 0$. This is intuitive because information can only be transmitted if an act can generate different prizes, or signal realizations, under a given prior. In turn, this departure from a degenerate act induces risk premium for risk averse DMs. The converse is not true since $\text{RP}(f) > 0$ can be the result of a constant but non-degenerate act, which is risky but uninformative, and hence $\text{IP}(f) = 0$.

3 Axioms and Representation Theorem

Axiom (Basic Axioms)**.**

1. *(Basic Rationality) The binary relation $\succsim$ is complete and transitive.*
2. *(Continuity) If $f \succ g \succ h$, then $\{a : af + (1-a)h \succsim g\}$ and $\{a : g \succsim af + (1-a)h\}$ are closed.*
3. *(Monotonicity) If $x > y$, then $\delta_x \succ \delta_y$.*[2]

[2]This can be replaced by a weaker, non-extremality condition (i.e., for all $x \in X$, neither $\delta_x \succsim \delta_y$

3.1 Skewness Independence

The benchmark model we depart from is Subjective Expected Utility (SEU), and its key axiom is the independence condition. The full scale of independence requires, indiscriminately, $f \succsim g \Leftrightarrow af + (1-a)\, h \succsim ag + (1-a)\, h$. The next axiom weakens this requirement by adding a prerequisite related to overlapping prizes. Let $f_x := (f(x \mid \omega_1), ..., f(x \mid \omega_N))$ capture the likelihood of prize x in different states. Given acts $h, f \in \mathcal{F}$, write $h_x \propto f_x$ if $kh_x = f_x$ for some $k \geq 0$. That is, the likelihood of prize x occurring through different states is proportional across the two acts (examples later).

Axiom (Skewness Independence)**.** *If $h_x \propto f_x$ and $h_x \propto g_x$ for all $x \in supp\,(h)$, then $f \succsim g \Leftrightarrow af + (1-a)\, h \succsim ag + (1-a)\, h$.*

A standard SEU maximizer considers only the aggregate probability of each prize occurring regardless of its potential skewness across states. Axiom 3.1 relaxes this condition by allowing for a DM who cares about whether a prize is more likely to occur through state ω_1 or through state ω_2. The mixture of overlapping acts can alter this likelihood ratio. In the following examples, the prizes of h do not overlap with the prizes of f and g, so independence holds between (f, g, h). For g' and g'', x_1 is now an overlapping prize with h. But $h_x \propto g'_x$, so independence should still hold between (f, g', h). Yet the mixture $\frac{1}{2}g'' + \frac{1}{2}h$ distorts the relative likelihood of x by a shift from ω_1 to ω_2, so the axiom allows independence to fail (i.e., $f \succ g''$ but $\frac{1}{2}g'' + \frac{1}{2}h \succ \frac{1}{2}f + \frac{1}{2}h$).

h	x	y
ω_1	$1/2$	$1/2$
ω_2	$1/2$	$1/2$

f	w	z
ω_1	1	0
ω_2	0	1

g	w	z
ω_1	0	1
ω_2	1	0

g'	x	z
ω_1	$1/4$	$3/4$
ω_2	$1/4$	$3/4$

g''	x	z
ω_1	$5/6$	$1/6$
ω_2	$1/6$	$5/6$

for all $y \in X$ nor $\delta_y \succsim \delta_x$ for all $y \in X$) along with the corresponding weakening of u in definition 1 and definition 2.

3.2 Prize Independence

Another standard axiom that is weakened is monotonicity. In most models, "upgrading" an act by replacing an inferior prize with a superior prize results in the new act deemed better. This assumption is sometimes called first-order stochastic dominance. Given act f, let fxp be the act that replaces prize x with lottery p.[3]

If $p \succsim x$, then fxp is an "upgrade" from f, and standard models predict $fxp \succsim f$. This is no longer obvious if the DM cares not only about the aggregate probability of a prize but also about its probability in each state. In particular, the replacement of x can distort the relative probability of y between states. To allow for monotonicity violations only in the face of these distortions, observe that distortions will not occur if $\text{supp}\,(p) \cap \text{supp}\,(f) = \emptyset$, because $f\,(x \mid \omega)\,p\,(y) > 0$ only if $f\,(y \mid \omega) = 0$. Hence we impose monotonicity that pertains to an upgrade with this precondition. For monotonicity that pertains to a downgrade, i.e., $x \succsim p \Rightarrow f \succsim fxp$, we impose without additional preconditions.

Axiom (Prize Independence)**.**

- *If $x \succsim p$ (resp. $\succ$), then $f \succsim fxp$ (resp. $\succ$).*
- *If $p \cap f = \emptyset$ and $p \succsim x$ (resp. $\succ$), then $fxp \succsim f$ (resp. $\succ$).*

Furthermore, if two prizes x, y are equally likely under every state, i.e., $f_x = f_y$, then equal distortions to both prizes should preserve indifferences. Formally, consider the weaker notion of replacing a prize where it is replaced only in a particular state. Given f, let $f_\omega xp$ be the result of replacing prize x with lottery p (only) under state ω.[4] We demand that two different ways of modifying f by replacing equally likely prizes with non-overlapping, indifferent lotteries do not make one modification better than another.

Axiom (State-Prize Independence)**.** *If $f_x = f_y$, $\delta_x \sim p$, $\delta_y \sim q$, $p \cap f = \emptyset$, and $q \cap f = \emptyset$, then $f_\omega xp \sim f_\omega yq$.*

[3] Formally, fxp yields the prize $y \neq x$ with the original probability of y plus the added probability of y due to the replacement of x, that is, $f\,(y \mid \omega) + f\,(x \mid \omega)\,p\,(y)$. It yields prize x with probability $f\,(x \mid \omega)\,p\,(x)$. So $fxp\,(y \mid \omega) = \begin{cases} f\,(y \mid \omega) + f\,(x \mid \omega)\,p\,(y) & y \neq x \\ f\,(x \mid \omega)\,p\,(x) & y = x \end{cases}$.

[4] $f_\omega xp\,(y \mid \omega) = \begin{cases} fxp\,(y \mid \omega') & \omega' = \omega \\ f\,(y \mid \omega') & \omega' \neq \omega \end{cases}$.

3.3 Mixture Aversion

The final axiom concerns the mixture of two indifferent acts. Under standard independence, the mixture of two acts is indifferent to the two acts themselves. The DMs we consider are mixture averse. Intuitively, the mixture operation tends to balance the relative probabilities of a prize under different states. Recall that Axiom 3.1 and Axiom 3.2 avoid imposing restrictions when these relative probabilities are potentially distorted. Mixture aversion focuses on a particular kind of reaction to these distortions, where making the same prize more-or-less likely to come from any state is disliked.

Axiom (Mixture Aversion)**.** *If* $f \sim g$*, then* $f \succsim af + (1-a)\, g$*, for any* $a \in [0,1]$.

To interpret, the DM prefers a strong association of each prize to a particular state, which may be viewed as a particular form of state-dependent preference for prizes. This is intuitive in the context of information acquisition, where it is well known that mixing two Blackwell experiments yields "balanced" likelihood ratios, thereby producing a posterior distribution that is closer to the prior, which is less valuable for the subsequent decision problem.

Many models of subjective risk focus on the exact opposite axiom called uncertainty aversion, where mixture is weakly preferred (i.e., $af + (1-a)\, g \succsim f$).[5] Gilboa and Schmeidler (1989) is most prominent, where mixtures are preferred and they underpin ambiguity averse preferences. In our setting, a certain form of uncertainty may be preferred because the resolution of these uncertainty contains useful information.

3.4 Representation Theorem

An intermediate representation bypasses the future menu A by directly specifying a value function V over posteriors:

Definition 2. $\succsim$ admits an **Informative Consumption - V (IC-V)** representation if there exist a full support prior $\mu \in \Delta(\Omega)$, a strictly increasing utility function

[5] Sarver (2018) studies a different version of mixture aversion where the primitive is preference over Epstein and Zin (1989) lotteries, and EU-violating behavior is understood as inconsistent risk attitudes.

$u : X \to \mathbb{R}$, a continuous and convex posterior value function $V : \Delta(\Omega) \to \mathbb{R}$ such that $\succsim$ is represented by

$$U(f) = U_C(f) + \sum_{x \in \text{supp}(f)} \underbrace{\sum_{\omega \in \Omega} \mu(\omega) f(x \mid \omega)}_{\text{probability of posterior } \mu_{x,f}} V(\mu_{x,f}).$$

Theorem 1. *$\succsim$ satisfies Basic Axioms, Skewness Independence, Prize Independence, and Mixture Aversion if and only if it admits an Informative Consumption (IC) - V representation.*

Theorem 2. *$\succsim$ admits an Informative Consumption (IC) - V representation (μ, u, V) where V is Lipschitz continuous if and only if it admits an Informative Consumption (IC) representation (μ, u, α, A).*[6]

We remark that the Lipschitz continuity of V can be obtained by adding a continuity axiom à la Dekel et al. (2007), where we essentially require that the equivalents of any two acts that are sufficiently close (in conditional probabilities) have bounded differences. We leave this attempt for future versions of this paper.

Identification

In both IC and IC-V, the prior μ is unique and the Bernoulli utility function u is unique up to positive affine transformation, as in standard SEU. For IC-V, the value function V can scale with u, which is standard. Moreover, because V can be "tilted" without changing the ex-ante value of any distribution of posteriors (i.e., the continuation value), we express this flexibility using the dot product $\lambda \cdot \pi$ in proposition 1.

Proposition 1. *Suppose $\succsim$ admits IC-V representation (μ_1, u_1, V_1), then it also admits an IC-V representation (μ_2, u_2, V_2) if and only if $\mu_1 = \mu_2$ and there exist $\kappa \in \mathbb{R}_{++}$, $\lambda \in \mathbb{R}^{|\Omega|}$, and $a, b \in \mathbb{R}$ such that $u_2 = \kappa u_1 + a$ and $V_2(\pi) = \kappa V_1(\pi) + \lambda \cdot \pi + b$ for all $\pi \in \Delta(\Omega)$.*

The identification of IC is affected by the inevitable non-uniqueness of the future menu A. For instance, the inclusion of dominated acts in A will not affect decisions. Moreover, multiplicity of optimal acts in A will induce redundancy. What

[6] Lipschitz continuous: there exists $L \geq 0$ such that for all $\pi, \pi' \in \Delta(\Omega)$, $|V(\pi) - V(\pi')| \leq L|\pi - \pi'|$.

can be identified, instead, is that admissible A's must induce the same posterior value function:

Definition 3. Given (μ, u, α, A), let the induced posterior value function $V^* : \Delta(\Omega) \to \mathbb{R}$ be

$$V^*(\mu_{x,f}) = \left(\frac{1-\alpha}{\alpha}\right) \sup_{g \in A} \sum_{\omega \in \Omega} \mu_{x,f}(\omega) \sum_{y \in \text{supp}(g)} g(y \mid \omega)\, u(y). \tag{3.1}$$

Proposition 2. *Suppose $\succsim$ admits IC representation $(\mu_1, u_1, \alpha_1, A_1)$, then it also admits an IC representation $(\mu_2, u_2, \alpha_2, A_2)$ if and only if $\mu_1 = \mu_2$ and there exist $\kappa \in \mathbb{R}_{++}$, $\lambda \in \mathbb{R}^{|\Omega|}$, and $a, b \in \mathbb{R}$ such that $u_2 = \kappa u_1 + a$ and $V_2^*(\pi) = \kappa V_1^*(\pi) + \lambda \cdot \pi + b$ for all $\pi \in \Delta(\Omega)$.*

Both identification results utilize the fact that every posterior belief is possible can be induced because μ has full support and $\mathcal{F}$ is the set of all acts/experiments.

4 Arrow–Debreu Information Coefficient

In general, the future menu A can involve arbitrarily many parameters. For applications and comparative statics, the most important feature of A may be a DM's degree of information incentive. We propose a one-dimensional measure that captures this incentive and its heterogeneity across DMs.

Assumption. *Throughout Section 4, u is continuous .*

4.1 Arrow–Debreu Menu

Drawing inspiration from the Arrow–Debreu asset, let $a_{\iota_i,\omega_i} \in \mathcal{F}$ be an act that pays $\iota_i \geq 0$ units of consumption in state ω_i and 0 in other states. We also normalize for $u(0) = 0$. We call A an Arrow–Debreu menu if $A = \{a_{\iota_1,\omega_1}, a_{\iota_2,\omega_2}, ..., a_{\iota_N,\omega_N}\}$. Given an Arrow–Debreu menu A, we call ι_i the coefficient of information incentive for state ω_i. A DM stands to gain more by choosing a_{ι_i,ω_i} if ι_i is greater, and a_{ι_i,ω_i} is the optimal act if the DM is sufficiently certain that the state is ω_i, hence ι_i indirectly captures the DM's incentive to distinguish state ω_i from other states. If the coefficients are the same across states, i.e., $\iota_i = \iota$ for all i, we say that A has a constant

Arrow–Debreu coefficient of information incentive ι. All else the same, a DM with a greater ι benefits more from information. Consider the following special case of IC.

Definition 4. We say $\succsim$ has an **Arrow–Debreu (AD)** representation (μ, u, ι) if it has an IC representation (μ, u, α, A) where A has a constant Arrow–Debreu coefficient of information incentive $u^{-1}\left(\frac{\alpha}{1-\alpha}u(\iota)\right)$.

When $\alpha = 0.5$, the coefficient of information incentive is simply ι. In general, the consumption weight α also affects a DM's information incentive, where a greater α (more weight on consumption) counterbalances a greater ι (more benefits from information). Therefore, and without loss of generality, the AD representation absorbs the effect of α into ι so that the single parameter ι captures total information incentive.

Proposition 3. *If $\succsim$ has an AD representation (μ, u, ι), then $\succsim$ is represented by $U(f) = U_C(f) + U_I(f)$ where*

$$U_I(f) = u(\iota)\,\mathbb{I}(f, \mu)$$

and

$$\mathbb{I}(f, \mu) \coloneqq \sum_{x \in supp(f)} \max_{\omega \in \Omega} \mu(\omega)\, f(x \mid \omega). \tag{4.1}$$

In particular, $U_I(f)$ (previously Equation 2.2) is now separable in (u, ι) and (f, μ) through a novel notion of *information index* $\mathbb{I}$, which has intuitive properties:[7]

1. $\mathbb{I}(f, \mu) = 1$ if f is fully informative.
2. $\mathbb{I}(f, \mu) \to 0$ if f is uninformative, μ is uniform, and $|\Omega| \to \infty$.
3. $\mathbb{I}(f, \mu) \geq \mathbb{I}(g, \mu)$ if f is Blackwell more informative than g.
4. $\mathbb{I}(f, \mu) > \mathbb{I}(g, \mu)$ if f is strictly Blackwell more informative than g.

If f is a constant act, then $\mathbb{I}(f, \mu)$ reduces to

$$\mathbb{N}(\mu) \coloneqq \max_{\omega \in \Omega} \mu(\omega). \tag{4.2}$$

[7] (1) $\mathbb{I}(f, \mu) = \sum_x \max_\omega \mu(\omega)\, \mathbb{1}_{x \sim \omega} = \sum_\omega \mu(\omega) = 1$. (2) $\mu(\omega) = \frac{1}{|\Omega|} \to 0$ so $\mathbb{I}(f, \mu) \to 0$. (3) and (4) are due to g being the product of a Markov kernel applied to f's signal realizations, which weakens the values attained by max.

A high $\mathbb{N}(\mu)$ corresponds to the situation where the DM is a priori confident about the true state.

Information incentive, subjective risk, information premium

The constant acts p_f and δ_z have the same information index $\mathbb{I}(f,\mu) = \mathbb{N}(\mu)$, so $U_I(p_f) = U_I(\delta_z) = u(\iota)\,\mathbb{N}(\mu)$. This is despite the fact that p_f may still contain objective risk. The result is $CE(p_f)$ simply solves $u(CE(p_f)) = U_C(p_f)$, so risk premium $\text{RP}(f)$ is never affected by information incentive ι.

The informative incentive ι influences $CE(f)$ through information premium, which in the AD representation simplifies to[8]

$$\text{IP}(f) = u^{-1}(U_C(f) + u(\iota)[\mathbb{I}(f,\mu) - \mathbb{N}(\mu)]) - u^{-1}(U_C(f)). \tag{4.3}$$

Note that information premium is increasing in the degree of information incentive ι, increasing in the information index $\mathbb{I}(f,\mu)$ of f, and decreasing in the DM's ex-ante confidence about the true state as measured by $\mathbb{N}(\mu)$, all of which are intuitive.

In the AD representation, if f is itself a constant act or the prior μ is degenerate, then $\mathbb{I}(f,\mu) = \mathbb{N}(\mu)$. Equation 4.3 then gives$\text{IP}(f) = 0$. Or if $\iota = 0$, then Equation 4.3 gives $\text{IP}(\cdot) = 0$. In other words, pure objective risk and the lack of information incentive each lead to standard SEU behavior, illustrating the discussion in Section 2.

Proposition 4. *For any given $f \in \mathcal{F}$, all else the same,*

1. $\text{RP}(f)$ *weakly increases when u is more risk-averse,*[9]
2. $\text{IP}(f)$ *weakly increases in ι,*
3. $\text{RP}(f) \geq \text{RP}(g)$ *if p_f is a mean-preserving spread of p_g,*
4. $\text{IP}(f) \geq \text{IP}(g)$ *if f Blackwell-dominates g and $CE(p_f) = CE(p_g)$.*

All weak notions become strict if the sufficient conditions become strict.

[8] By solving $U_I(\delta_{CE(f)}) = u(z) + u(\iota)(\max_\omega \mu(\omega)) \equiv \sum_x p_f(x)\,u(x) + u(\iota)\,\mathbb{I}(f,\mu) = U_I(f)$ for $CE(f) = u^{-1}(U_C(f) + u(\iota)[\mathbb{I}(f,\mu) - \mathbb{N}(\mu)])$.

[9] As in standard, we say u is more risk averse than v if there exists a concave $h : \mathbb{R} \to \mathbb{R}$ such that $u = h \circ v$.

Proof. Items 1 and 3 are well-known. Items 2 and 4 are straightforward from Equation 4.3 and the properties of $\mathbb{I}$. □

The special case of risk neutrality (i.e., $u(x) = \lambda x$, $\lambda > 0$) simplifies Equation 4.3 to

$$\text{IP}(f) = \iota\left[\mathbb{I}(f, \mu) - \mathbb{N}(\mu)\right],$$

which is independent of the exact prizes f sends.

4.2 Arrow–Debreu–Pratt Representation

It is much more common in applications to work with a parameterized Bernoulli utility function like the one associated with the constant absolute risk aversion,

$$u_\rho(x) = 1 - e^{-\rho x},$$

where ρ is the (constant) Arrow–Pratt coefficient of absolute risk aversion.

Paired with our constant Arrow-Debreu coefficient of information incentive ι, we obtain a 2-parameter model given in definition 5. Clearly, the results and observations about AD representations apply.

Definition 5. We say $\succsim$ has an **Arrow–Debreu–Pratt (ADP)** representation (μ, ρ, ι) if it has an AD representation (μ, u_ρ, ι).

In standard expected utility, the coefficient ρ provides a simple estimate of an individual's risk aversion. A higher ρ is always associated with a higher risk premium; that is, the individual is willing to accept a lower sure prize (certainty equivalent) to give up the same act, holding prior fixed. Therefore, if two individuals with the same prior have different certainty equivalents for the same act, it must be that the individual with a lower certainty equivalent has a greater ρ, i.e., is more risk averse.

This is no longer true. Rewriting Equation 2.4 with preference parameters included (holding μ fixed),

$$\begin{aligned} CE_{\rho_1,\iota_1}(f) &= \mathbb{E}_f(x) - \text{RP}_{\rho_1}(f) + \text{IP}_{\rho_1,\iota_1}(f) \\ CE_{\rho_2,\iota_2}(f) &= \mathbb{E}_f(x) - \text{RP}_{\rho_2}(f) + \text{IP}_{\rho_2,\iota_2}(f). \end{aligned}$$

Note that $CE_{\rho_1,\iota_1}(f) = CE_{\rho_2,\iota_2}(f)$ can be consistent with $\rho_1 > \rho_2$. In that case, $\text{RP}_{\rho_1}(f) > \text{RP}_{\iota_2}(f)$ but $\text{IP}_{\rho_1,\iota_1}(f) > \text{IP}_{\rho_2,\iota_2}(f)$.

That is, even though individual 1 is more risk averse than individual 2, which should lead to the willingness to accept a lower sure prize, she also benefits more from information relative to individual 2 and therefore sacrifices more from accepting a sure prize. When the two opposite forces exactly cancel each other out, her certainty equivalent is the same as individual 2's.

More generally, if an individual takes less risk by choosing g over f and another individual takes more risk by choosing f over g, standard methods infer that the former is more risk averse. What we emphasize is that the two individuals could well have the same risk aversion $\rho_1 = \rho_2$ but differ in their information incentive $\iota_1 \neq \iota_2$. It turns out that the information index $\mathbb{I}$ (Equation 4.1) underpins this indeterminacy.

Let $\succsim_{(\mu,\rho,\iota)}$ denote the preference under an ADP representation (μ, ρ, ι) and $\succ_{(\mu,\rho,\iota)}$ its asymmetric component (strict preference). Recall that $\succsim_{(\mu,\rho,0)}$ reduces to standard SEU.

Proposition 5. *Suppose*

$$g \succ_{(\mu,\rho_1,0)} f \qquad f \succ_{(\mu,\rho_2,0)} g$$

for some $\rho_1 > \rho_2$*. There exist* $\bar{\rho}$ *and* $\iota_1 < \iota_2$ *such that*

$$g \succ_{(\mu,\bar{\rho},\iota_1)} f \qquad f \succ_{(\mu,\bar{\rho},\iota_2)} g$$

if and only if $\mathbb{I}(f, \mu) > \mathbb{I}(g, \mu)$.

Joint identification of parameters

proposition 5 highlights potential indeterminacy in preference parameters when an act has both greater aggregate risk and a greater information index. Joint identification is still possible; just not with these acts. Counter-intuitively at first, using acts that do not "dominate" one another does not necessarily provide identification, because different combinations of risk aversion and information incentive can cancel each other out.

Joint identification of ρ and ι can be achieved by carefully separating objective risk and subjective risk. In what follows, we compare two constant acts to identify ρ, compare two acts with the same degree of information to identify μ, and compare

a fully informative act with a constant act to identify ι.

Recall that constant acts have an information premium of 0. Therefore, if g is a non-degenerate constant act and the elicited certainty equivalent is z, then ρ^* simply solves $\mathbb{E}_{p_g}\left(u_\rho\left(x\right)\right) = u_\rho\left(g\right)$, where $p_g = g\left(\omega_1\right)$ is independent of the prior.

Next, suppose $x_1, x_2 > 0$ and the DM is found indifferent between h and h_z.

h	x_1	x_2
ω_1	1	0
ω_2	0	1

$h_{z'}$	z'	0
ω_1	1	0
ω_2	0	1

Since both are fully informative ($\mathbb{I}\left(h, \mu\right) = \mathbb{I}\left(h_{z'}, \mu\right) = 1$), μ^* uniquely solves

$$\mu^*\left(\omega_1\right) u_{\rho^*}\left(x_1\right) + \left(1 - \mu^*\left(\omega_1\right)\right) u_{\rho^*}\left(x_2\right) \equiv \mu^*\left(\omega_1\right) u_{\rho^*}\left(z'\right).$$

If there are more than two states, a workaround that maintains $\mathbb{I}\left(h, \mu\right) = \mathbb{I}\left(h_{z'}, \mu\right)$ is to assign a third distinct prize to both h and $h_{z'}$ for every other state. This identifies $\mu\left(\omega_2\right)/\mu\left(\omega_1\right)$. The procedure can be repeated for every $\mu\left(\omega_i\right)/\mu\left(\omega_1\right)$ to eventually identify μ.[10]

Finally, let f be a fully informative act and suppose its elicited certainty equivalent is z'', then ι^* uniquely solves

$$u_{\rho^*}\left(z''\right) + u_{\rho^*}\left(\iota\right)\mathbb{N}\left(\mu^*\right) \equiv \mathbb{E}_{p_f}\left(u_{\rho^*}\left(x\right)\right) + u_{\rho^*}\left(\iota\right)\mathbb{I}\left(f, \mu^*\right)$$

(proposition 3) where $\mathbb{I}\left(f, \mu\right) = 1$ because f is fully informative and p_f is the lottery reduction of f under μ^*.

5 Conclusion

The focus of this paper is the inference of a DM's *subjective future menu*, which entails the DM's perception of future problems and the need to sacrifice consumption today for information tomorrow.

Our axiomatic characterization exploits the behavioral signatures of a DM's preferences over Anscombe–Aumann acts—which yield objective information upon

[10] For instance if $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, then $h\left(\omega_3\right) = h_{z'}\left(\omega_3\right) = h\left(\omega_4\right) = h_{z'}\left(\omega_4\right) = \delta_{x_3}$, so $\mathbb{I}\left(h, \mu\right) = \mu\left(\omega_1\right) + \mu\left(\omega_2\right) + \max_{\tilde{\omega}\in\{\omega_3,\omega_4\}} \mu\left(\tilde{\omega}\right)$ and $\mathbb{I}\left(h_{z'}, \mu\right) = \mu\left(\omega_1\right) + \mu\left(\omega_2\right) + \max_{\tilde{\omega}\in\{\omega_3,\omega_4\}} \mu\left(\tilde{\omega}\right)$, so we are back to $U_I\left(h\right) = U_I\left(h_{z'}\right)$.

the resolution of uncertainty—to model this consumption–information trade-off. We introduce several analytical tools to study information incentives, including a novel notion of an information premium (analogous to a risk premium) and a two-parameter ADP parameterization that independently captures risk aversion and information incentives.

From an alternative perspective, the model can be interpreted as a framework for information acquisition with a consumption cost. This forgone consumption utility provides a microfoundation for why information is inherently costly. In ongoing work, we explore an extension of this model where—because consumption derives from expected utility and beliefs converge more slowly toward the extremes—pairing a linear consumption loss with diminishing information gains naturally gives rise to convex information costs.

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A Appendix: Proofs

A.1 Proof of Theorem 1

The **"if"** direction is straightforward except possibly for Axiom 3.3: Suppose that $\succsim$ admits an IC-V representation (μ, u, V). Denote the aggregate probability of $x \in X$ under $f \in \mathcal{F}$ by $p_f(x) := \sum_{\omega\in\Omega} \mu(\omega) f(x \mid \omega)$. Fix any two acts $f, g \in \mathcal{F}$ and $\alpha \in (0,1)$. Let $h := \alpha f + (1-\alpha) g$. For any $x \in \text{supp}(h)$ and any $\omega \in \Omega$, since $\mu_{x,h}(\omega) = \frac{\mu(\omega)h(x|\omega)}{p_h(x)}$ and $h(x \mid \omega) = \alpha f(x \mid \omega) + (1-\alpha) g(x \mid \omega)$, we have

$$\begin{aligned}
\mu_{x,h}(\omega) &= \frac{\mu(\omega)\left[\alpha f(x|\omega) + (1-\alpha) g(x|\omega)\right]}{p_h(x)} \\
&= \frac{\alpha p_f(x)}{p_h(x)} \frac{\mu(\omega) f(x|\omega)}{p_f(x)} + \frac{(1-\alpha) p_g(x)}{p_h(x)} \frac{\mu(\omega) g(x|\omega)}{p_g(x)} \\
&= \frac{\alpha p_f(x)}{p_h(x)} \mu_{x,f}(\omega) + \frac{(1-\alpha) p_g(x)}{p_h(x)} \mu_{x,g}(\omega).
\end{aligned}$$

It can be checked that

$$\frac{\alpha p_f(x)}{p_h(x)} + \frac{(1-\alpha) p_g(x)}{p_h(x)} = \frac{\sum_{\omega\in\Omega} \mu(\omega)\left[\alpha f(x \mid \omega) + (1-\alpha) g(x|\omega)\right]}{\sum_{\omega\in\Omega} \mu(\omega) h(x \mid \omega)} = 1.$$

Since V is convex, Jensen's inequality gives

$$V(\mu_{x,h}) \leq \frac{\alpha p_f(x)}{p_h(x)} (V \mu_{x,f}) + \frac{(1-\alpha) p_g(x)}{p_h(x)} V(\mu_{x,g}).$$

Multiplying by $p_h(x)$ and summing over $x \in \text{supp}(h)$ gives

$$\sum_{x\in\text{supp}(h)} p_h(x) V(\mu_{x,h}) \leq \alpha \sum_{x\in\text{supp}(f)} p_f(x) V(\mu_{x,f}) + (1-\alpha) \sum_{x\in\text{supp}(g)} p_g(x) V(\mu_{x,g}).$$

And utility is $U_C(h) = \alpha U_C(f) + (1-\alpha) U_C(g)$ due to SEU. Therefore, we have $U(h) \leq \alpha U(f) + (1-\alpha) U(g)$ as desired.

We also remark that Continuity in Axiom 3 is due to both U_C and V are continuous (in conditional probabilities of acts).

We prove **"only if"** next.

Step 0: Notation and objects

For any $f \in \mathcal{F}$, we say that $f_x^* \in \mathcal{F}$ is a *spitting of prize* $x \in \text{supp}(f)$ if f_x^* can be obtained through the following operation: There is a set of lotteries $\{p_1, ..., p_{\{\Omega\}}\}$ satisfying $\text{supp}(p_i) \cap \text{supp}(f) = \emptyset$ for all i, $\text{supp}(p_i) \cap \text{supp}(p_j) = \emptyset$ for all i, j, and $p_i \sim x$ for all i, such that

$$f_x^*(z \mid \omega_i) \coloneqq \begin{cases} f(x \mid \omega_i) p_i(z) & z \in \text{supp}(p_i) \\ f(z \mid \omega_i) & z \neq x \end{cases}.$$

In words, f_x^* is obtained by assigning a non-overlapping state-dependent lottery replacement of prize x in f. Given $f \in \mathcal{F}$ and $x \in X$, Let

$$F^*(f, x)$$

be the set of all f_x^*. Note that $F^*(f, x) \neq \emptyset$ because $\text{supp}(f)$ is finite and $X = \mathbb{R}$. Moreover, note that if f_x is such that for some $\omega_i \in \Omega$ we have $f_x(\omega_i) > 0$ and $f_x(\omega_j) = 0$ for all $\omega_j \in \Omega \backslash \{\omega_i\}$, then $f \in F^*(f, x)$. For any x, $f_x^* \in F^*(f, x)$, and $f_x^{*\prime} \in F^*(f', x)$, we say (f_x^*, f_x^*) *involve common splits* if their operations involve the same $\{p_1, ..., p_N\}$. If $x \notin \text{supp}(f)$, then let $F^*(f, x) = \{f\}$.

For any $f \in \mathcal{F}$, define $f_I^* \in \mathcal{F}$ as follows: Turn $\text{supp}(f)$ into an ordered set $\{x_1, ..., x_M\} \subset X$. For the first prize x_1, pick f^1 from $F^*(f, x_1)$. If $M > 1$, then for the second prize x_2, pick f^2 from $F^*(f^1, x_2)$ satisfying $\text{supp}(f^2) \cap \{x_1\} = \emptyset$. If $M > 2$, iterate this process for each other i to obtain $f^3, ..., f^M$, each satisfying $\text{supp}(f^i) \cap \{x_1, ..., x_{i-1}\} = \emptyset$, which is possible at every step because $\text{supp}(f^i)$ is finite and $X = \mathbb{R}$. Finally, let $f_I^* \coloneqq f^M$. In general, f_I^* is not unique (given f) because the process of picking $f^1, f^2, ...$ involve freedom. Let

$$I^*(f)$$

denote the set of all f_I^*. Note that $I^*(f) \neq \emptyset$ because $\text{supp}(f)$ is finite and $X = \mathbb{R}$.

If an act $f \in \mathcal{F}$ is such that for any $x \in \text{supp}(I)$, there exists $\omega_i \in \Omega$ such that $f(x \mid \omega_i) > 0$ and $f(x \mid \omega_j) = 0$ for all $\omega_j \neq \omega_i$, then we call it a (fully) *informative act*. Let $\mathcal{F}_I$ denote the set of all informative acts, and its typical element is denoted by I. Clearly, $I^*(f) \subset \mathcal{F}_I$ for all f.

If an act $f \in \mathcal{F}$ is such that for all $\omega_i \in \Omega$, $f(x_i \mid \omega_i) = 1$ for some $x_i \in X$, and $x_i \neq x_j$ for all $i \neq j$, then we call it a *simple act*. Let $\mathcal{F}_S$ denote the set of all simple acts, and its typical element is denoted by f^s. Clearly, $\mathcal{F}_S \subset \mathcal{F}_I$.

Let $\mathcal{F}_P \subset \mathcal{F}$ denote the set of constant acts. Let $\mathcal{F}_X \subset \mathcal{F}$ denote the set of degenerate constant acts.

Step 1: Construction of U and its properties

Lemma 1. *If $f \succ g$ and $f_x \propto g_x$ for all $x \in$ supp (f), then $af + (1-a)g \succ \beta f + (1-\beta)g$ for all $a, \beta \in [0,1]$ such that $a > \beta$.*

Proof. Fix qualifying f, g, a, β. By Axiom 3.1, $kf + (1-k)g \succ kg + (1-k)g$ for $k = \frac{a-\beta}{1-\beta}$, then since $(kf + (1-k)g)_x \propto f_x$ for any $x \in$ supp (f), by Axiom 3.1 again, $af + (1-a)g = \beta f + (1-\beta)(kf + (1-k)g) \succ \beta f + (1-\beta)g$. □

Claim 1. For any $f \in \mathcal{F}$, there exist a degenerate prize $\delta_x \in \mathcal{F}$ and a simple act $g \in \mathcal{F}_S$ such that $g \succsim f \succsim \delta_x$.

Proof. Fix $f \in \mathcal{F}$. We first show there exists a degenerate prize $\delta_x \in \mathcal{F}$ such that $f \succsim \delta_x$. Let x be $\min_{\succeq}$ supp (f). We iteratively replacing other prizes in the support of f with prize x using the following process. Pick $y_1 \in$ supp $(f) \backslash \{x\}$. Let $f^1 = fy_1x$. Therefore, by prize independence, since $y \succsim x$, we have $f \succsim f^1$. Pick $y_2 \in$ supp $(f^1) \backslash \{x\}$ and let $f^2 = f^1 y_2 x$. Since $y_2 \succsim x$, then by prize independence we have $f^1 \succsim f^2$. By iteratively generating f^i, we finally have $f^N = \delta_x$, where $N = |\text{supp}(f)| - 1$. Then $f \succsim f^1 \succsim f^2 \succsim ... \succsim f^N$. Therefore $f \succsim \delta_x$.

We next show there exist a simple act $g \in \mathcal{F}_S$ such that $g \succsim f$. Let $\mathcal{F}(f) = \{h \in \mathcal{F} : \forall \omega \in \Omega, \exists x \in \text{supp}(f(\omega)) \, s.t. \, h(x|\omega) = 1\}$. Then there exists a convex combination over $\mathcal{F}(f)$ equivalent to f. Denote this convex combination by $\sum_{f^i \in \mathcal{F}(f)} a_i f^i$, where $a_i \geq 0, \sum a_i = 1$. For every $i \in |\mathcal{F}(f)|$, pick a simple act f^i_s such that $f^i_s \cap f^j_s = \emptyset$ for any $j \neq i$ and for any $x \in$ supp (f^i_s), $\delta_x \succsim \delta_y$ for all $y \in$ supp (f). We can do the combination of simple acts because the prize space is a real line. Denote $\sum_{i \in |\mathcal{F}(f)|} a_i f^i_s$ as h. Notice that h is an informative act. We can obtain $\sum_{f^i \in \mathcal{F}(f)} a_i f^i$ from h by the following process. For every $i \in |\mathcal{F}(f)|$, replace $f^i_s(\omega)$ with $f^i(\omega)$ for any $\omega \in \Omega$. And since $f^i_s(\omega) \succsim f^i(\omega)$ for any $\omega \in \Omega$, by prize independence we have $h \succsim \sum_{f^i \in \mathcal{F}(f)} a_i f^i$, i.e., $h \succsim f$. Then, there exists a simple act $g \in \mathcal{F}_S$ such that for any $x \in$ supp(g), $x \succsim y$ for any $y \in$ supp (h). By Axiom 3.1, we

have $g(\omega) \succsim h(\omega)$ for any $\omega \in \Omega$. Then by iteratively replacing $g(\omega_i)$ with $h(\omega_i)$ for every $i \in |\Omega|$, we can obtain h from g. Therefore, by Axiom 3.2, since $g(\omega) \succsim h(\omega)$ for any $\omega \in \Omega$, we have $g \succsim h$. Therefore, $g \succsim h \succsim f$. □

Lemma 2. *There exists a utility function $U : \mathcal{F} \to \mathbb{R}$ such that* the following hold:

1. For any $f, g \in \mathcal{F}$, $f \succsim g$ if and only if $U(f) \geq U(g)$.
2. For any $f, g \in \mathcal{F}$ such that $f_x \propto g_x$ for any $x \in$ *supp* (f), $U(af + (1-a)g) = aU(f) + (1-a)U(g)$ for all $a \in [0,1]$.
3. *If supp* $(f) \cap$ *supp* $(g) = \emptyset$, *then* $U(af + (1-a)g) = aU(f) + (1-a)U(g)$ for all $a \in [0,1]$.

Proof. Notice that Axiom 3.1 includes the standard independence axiom on constant acts, that is, $p \succsim q \Leftrightarrow ap + (1-a)r \succsim aq + (1-a)r$ for all constant acts $p, q, r \in \mathcal{F}_P$ and $a \in [0,1]$. Fix constant, degenerate acts $\delta_b, \delta_0 \in \mathcal{F}$ such that $\delta_b \succ \delta_0$, there exists a utility function $\bar{U} : \Delta(X) \to \mathbb{R}$ such that $\bar{U}(\delta_0) = 0$, $\bar{U}(\delta_b) = 1$, and for any $p, q \in \mathcal{F}_P$, $\bar{U}(ap + (1-a)q) = a\bar{U}(p) + (1-a)\bar{U}(q)$.

Define $f \propto g$ if $f_x \propto g_x$ for any $x \in \text{supp}(f)$. Let

$$\mathcal{F}_{b,0} := \{f : \delta_b \succsim f \succsim \delta_0, f \propto \delta_b, f \propto \delta_0\}.$$

For every $f \in \mathcal{F}_{b,0}$, Continuity in Axiom 3 and lemma 1 together guarantee a unique number $\lambda_f \in [0,1]$ such that $\lambda_f \delta_b + (1-\lambda_f)\delta_0 \sim f$; set $U_0(f) = \lambda_f$. Note that for all $f, g \in \mathcal{F}_{b,0}$, Weak Order in Axiom 3 gives $f \succsim g \Leftrightarrow U_0(f) \geq U_0(g)$. Note also that $U_0(\delta_0) = \bar{U}(\delta_0) = 0$ and $U_0(\delta_b) = \bar{U}(\delta_b) = 1$, so $U_0(p) = \bar{U}(p)$ for any $p \in \mathcal{F}_P \cap \mathcal{F}_{b,0}$. We have constructed $U_0 : \mathcal{F}_{b,0} \to \mathbb{R}$.

Next, consider any $f, g \in \mathcal{F}_{b,0}$ such that $f \propto g$. We show that for any $a \in [0,1]$, $U_0(af + (1-a)g) = aU_0(f) + (1-a)U_0(g)$.

Since $f, g \in \mathcal{F}_{b,0}$ implies $f, g \succsim \delta_0$, Axiom 3.1 gives $af + (1-a)g \succsim a\delta_0 + (1-a)g$ and $a\delta_0 + (1-a)g \succsim a\delta_0 + (1-a)\delta_0$, then Weak Order in Axiom 3 gives $af + (1-a)g \succsim \delta_0$. Similarly, $\delta_b \succsim f, g$ leads to $\delta_b \succsim af + (1-a)g$. So $af + (1-a)g \in \mathcal{F}_{b,0}$.

Fix any $f, g \in \mathcal{F}_{b,0}$. By U_0's definition, $U_0(f)\delta_b + (1 - U_0(f))\delta_0 \sim f$ and $U_0(g)\delta_b + (1 - U_0(g))\delta_0 \sim g$. Since we have $f_z \propto (\delta_0)_z$, $f_z \propto p_z$, and $f_z \propto (\delta_0)_z$ for all

$z \in \text{supp}(f)$, multiple applications of Axiom 3.1 give

$$\begin{aligned}
&af + (1-a)\, g \\
&\sim af + (1-a)\left(U_0(g)\,\delta_b + (1 - U_0(g))\,\delta_0\right) \\
&\sim a\left[U_0(f)\,\delta_b + (1 - U_0(f))\,\delta_0\right] + (1-a)\left[U_0(g)\,\delta_b + (1 - U_0(g))\,\delta_0\right]
\end{aligned}$$

for all $a \in [0,1]$. Note that the last line is constant act, on which the affine property of U_0 has already been established (when $\bar{U}$ was defined), so

$$\begin{aligned}
&U_0(af + (1-a)\, g) \\
&= U_0\left(a\left[U_0(f)\,\delta_b + (1 - U_0(f))\,\delta_0\right] + (1-a)\left[U_0(g)\,\delta_b + (1 - U_0(g))\,\delta_0\right]\right) \\
&= aU_0\left(U_0(f)\,\delta_b + (1 - U_0(f))\,\delta_0\right) + (1-a)\,U_0\left(U_0(g)\,\delta_b + (1 - U_0(g))\,\delta_0\right) \\
&= aU_0(f) + (1-a)\,U_0(g)
\end{aligned}$$

for all $a \in [0,1]$.

We have established the two properties in lemma 2 on $U_0 : \mathcal{F}_{b,0} \to \mathbb{R}$. Next we extend these properties to all of $\mathcal{F}$. Let

$$\mathcal{F}_{f_i,g_i} \coloneqq \left\{h \in \mathcal{F} : f_i \succ h \succ g_i, f_i \propto h, g_i \propto h\right\}.$$

For every $i \in \mathbb{N}_+$, pick $f_i \in \mathcal{F}_S$ and $g_i \in \mathcal{F}_X$ such that the following conditions hold: (1) $\{\delta_b, \delta_0\} \cap (\text{supp}(f_i) \cup \text{supp}(g_i)) = \emptyset$ for all $i \in \mathbb{N}_+$, (2) $f_i \succ f_{i-1} \succ g_{i-1} \succ g_i$ for all $i \in \mathbb{N}_+$. By claim 1, for any $h \in \mathcal{F}$, there exists $k \in \mathbb{N}$ such that $h \in \mathcal{F}_{f_k,g_k}$.

Fix any $i \in \mathbb{N}_+$. For every $h \in \mathcal{F}_{f_i,g_i}$, Continuity in Axiom 3 and lemma 1 together guarantee a unique number $\lambda_h \in [0,1]$ such that $\lambda_f f_i + (1-\lambda_f)\, g_i \sim h$; set $U_i^*(h) = \lambda_h$. Note that using the same line of arguments as before, $U_i^*(f) \geq U_i^*(g) \Leftrightarrow f \succsim g$ for all $f, g \in \mathcal{F}_{f_i,g_i}$, and $U_i(af + (1-a)\, g) = aU_i(f) + (1-a)\,U_i(g)$ for all $f, g \in \mathcal{F}_{f_i,g_i}$ and $a \in [0,1]$. Let U_i be a positive affine transformation of U_i^* so that $U_i(\delta_b) = 1$ and $U_i(\delta_0) = 0$. For later, we observe that for any degenerate prize $\delta_x \in \mathcal{F}_{f_i,g_i}$, we have $U_i(\delta_x) = \bar{U}(\delta_x)$ for every $i \in \mathbb{N}_+$.

Fix any $h \in \mathcal{F}_{f_i,g_i} \cap \mathcal{F}_{f_j,g_j}$. Fix $x, y \in X$ such that $\delta_x \succ \delta_y$, $\{\delta_x, \delta_y\} \subset \left(\mathcal{F}_{f_i,g_i} \cap \mathcal{F}_{f_j,g_j}\right)$, and $\text{supp}(h) \cap \{x, y\} = \emptyset$; their existence is guaranteed because $\mathcal{F}_{\delta_b,\delta_0} \subset \mathcal{F}_{f_i,g_i}$ and $\mathcal{F}_{\delta_b,\delta_0}$ contains uncountably many elements of X. By lemma 1

and Continuity in Axiom 3, there are three possible cases:

$$\begin{aligned}\delta_x \succsim h \succsim \delta_y, h \sim a\delta_x + (1-a)\,\delta_y,\\ \delta_x \succ \delta_y \succsim h, \delta_y \sim a\delta_x + (1-a)\,h,\\ h \succsim \delta_x \succ \delta_y, \delta_x \sim ah + (1-a)\,\delta_y,\end{aligned}$$

where $a \in [0,1]$ is unique. Since we have established that for all $k \in \mathbb{N}$, U_k represent preference on $\mathcal{F}_{f_k,g_k}$ and U_k has affine structure for any $f, g \in \mathcal{F}_{f_k,g_k}$ such that $f \propto g$, then, for each the above possibilities, we have for $k \in \{i, j\}$,

$$\begin{aligned}U_k(f) &= aU_k(\delta_x) + (1-a)\,U_k(\delta_y),\\ U_k(\delta_y) &= aU_k(\delta_x) + (1-a)\,U_k(h),\\ U_k(\delta_x) &= aU_k(h) + (1-a)\,U_k(\delta_y),\end{aligned}$$

respectively. Since $U_k(\delta_x) = U_0(\delta_x)$ and $U_k(\delta_y) = U_0(\delta_y)$ for all $k \in \mathbb{N}$, so $U_i(h) = U_j(h)$ regardless of which of the three possible cases is true.

Finally, let $U : \mathcal{F} \to \mathbb{R}$ be the common value of U_i. Assured by the above arguments, U is well-defined. Fix any $f, g \in \mathcal{F}$, note that there exists $\mathcal{F}_{f_i,g_i}$ such that $f, g \in \mathcal{F}_{f_i,g_i}$. Note that

$$U(f) = U_i(f) \geq U_i(g) = U(g) \Leftrightarrow f \succsim g,$$

so have shown property 1 in lemma 2. Moreover, if $f_x \propto g_x$ for all $x \in \operatorname{supp}(f)$, then

$$U(af + (1-a)\,g) = U_i(af + (1-a)\,g) = aU_i(f) + (1-a)\,U_i(g) = aU(f) + (1-a)\,U(g)$$

for all $a \in [0,1]$, so we have shown property 2 in lemma 2. Property 3 in lemma 2 is simply because $\operatorname{supp}(f) \cap \operatorname{supp}(g) = \emptyset$ implies $f_x \propto g_x$ for any $x \in \operatorname{supp}(f)$. □

Claim 2. For any $f \in \mathcal{F}$ and $f_x^*, f_x^{*\prime} \in F^*(f, x)$, $U(f_x^*) = U(f_x^{*\prime})$.

Proof. This is straightforward by multiple applications of Axiom 3.2. □

Claim 3. For any $f \in \mathcal{F}$ and $f_I^*, f_I^{*\prime} \in I^*(f)$, $U(f_I^*) = U(f_I^{*\prime})$.

Proof. By applying claim 2 iteratively. □

Step 2: Construction of v bar and its properties

Let the function $\bar{v} : [0,1]^N \to \mathbb{R}$ satisfy

$$\bar{v}(f_x) = U(f_x^*) - U(f)$$

where $f_x^* \in F^*(f,x)$ (defined in Subsection A.1). Note that $\bar{v}(\mathbf{0}) = 0$ because $f_x = \mathbf{0}$ implies $x \notin \text{supp}(f)$, which implies $F^*(f,x) = \{f\}$ by definition. Moreover, note that if f_x is such that for some $\omega_i \in \Omega$, $f_x(\omega_i) > 0$ and $f_x(\omega_j) = 0$ for all $\omega_j \in \Omega \backslash \{\omega_i\}$, then $f \in F^*(f,x)$, so $\bar{v}(f_x) = 0$.

Claim 4. $\bar{v}$ is well-defined.

Proof. For any $f \in \mathcal{F}$ and $x \in \text{supp}(f)$, let

$$\mathcal{F}_1(f,x) \coloneqq \{h \in \mathcal{F} : h_x = f_x, h_y = \mathbf{1}_N - f_x, \exists y \neq x\}$$
$$\mathcal{F}_2(f,x) \coloneqq \{h \in \mathcal{F} : h_z = f_x, h_{z'} = \mathbf{1}_N - f_x, \exists z, z' \in X\}.$$

To show that $\bar{v}(f_x)$ is well-defined, we need to establish two statements.

1. For any $f \in \mathcal{F}$ and $x \in \text{supp}(f)$, if $h \in \mathcal{F}_1(f,x)$, then $U(f_x^*) - U(f) = U(h_x^*) - U(h)$.
2. For any $f \in \mathcal{F}$, $x \in \text{supp}(f)$, and $h \in \mathcal{F}_2(f,x)$, if $h_z = f_x$, then $U(f_x^*) - U(f) = U(h_z^*) - U(h)$.

To prove statement 1, fix any $f \in \mathcal{F}$ and $x \in \text{supp}(f)$. Consider any $h \in \mathcal{F}_1(f,x)$ such that $\text{supp}(f) \cap \text{supp}(h) = \{x\}$. Fix any $h_x^* \in F^*(h,x)$ such that $\text{supp}(f) \cap \text{supp}(h_x^*) = \{x\}$ and (f_x^*, h_x^*) involve common splits (Subsection A.1). By definition of split, $\frac{1}{2}f_x^* + \frac{1}{2}h = \frac{1}{2}h_x^* + \frac{1}{2}f$. Since $(f_x^*)_z \propto h_z$ for all $z \in \text{supp}(h)$ and $(h_x^*)_z \propto f_z$ for all $z \in \text{supp}(f)$, so lemma 2 gives the first and last equalities in

$$\frac{1}{2}U(f_x^*) + \frac{1}{2}U(h) = U\left(\frac{1}{2}f_x^* + \frac{1}{2}h\right) = U\left(\frac{1}{2}h_x^* + \frac{1}{2}f\right) = \frac{1}{2}U(h_x^*) + \frac{1}{2}U(f).$$

So we have $U(f_x^*) - U(f) = U(h_x^*) - U(h)$. Fix one such h, which exists because $\text{supp}(f)$ is finite and $X = \mathbb{R}$. Consider any $h' \in \mathcal{F}_1(f,x)$ such that $\text{supp}(f) \cap \text{supp}(h') = \{x,z\}$ for some $z \in X$ and $\text{supp}(h) \cap \text{supp}(h') = \{x\}$. Consider

$h_x^* \in F^*(h,x)$ and $h_x'^* \in F^*(h',x)$ such that $(h_x^*, h_x'^*)$ involve common splits. Then we have $\frac{1}{2}h_x'^* + \frac{1}{2}h = \frac{1}{2}h_x^* + \frac{1}{2}h'$, and therefore

$$U(f_x^*) - U(f) = U(h_x^*) - U(h) = U(h_x'^*) - U(h')$$

using a similar argument as before. Note that every act in $\mathcal{F}_1(f,x)$ has a support of at most size two, so we have covered all of $\mathcal{F}_1(f,x)$.

To prove statement 2, fix any $f_0 \in \mathcal{F}$ such that $x \in \text{supp}(f_0)$. Pick $f \in \mathcal{F}_1(f_0,x)$ such that $f_x = (f_0)_x$ and $\text{supp}(f) = \{x,z\}$. Consider any $h \in \mathcal{F}_2(f_0,x)$ such that $h_y = f_x$ for some $y \neq x$ and $h_z = f_z$. Let $\bar{h} := \frac{1}{2}f + \frac{1}{2}h$, then $\bar{h}_x^* = \frac{1}{2}f_x^* + \frac{1}{2}h$ and $\bar{h}_y^* = \frac{1}{2}h_y^* + \frac{1}{2}f$. Consider the constant act $p_1 \in \mathcal{F}_P$ such that $p_1 \sim \delta_x$ and $\text{supp}(\bar{h}) \cap \text{supp}(p_1) = \emptyset$. Let $\bar{h}_{x1} = \bar{h}_{\omega_1 x p_1}$. Consider $p_2 \in \mathcal{F}_P$ such that $p_2 \sim \delta_x$ and $\text{supp}(\bar{h}_{x1}) \cap \text{supp}(p_2) = \emptyset$. Let p_2 replace x under ω_1 in $\bar{h}_{x1}$, denote the new act $\bar{h}_{x1\omega_1 x p_2}$ by $\bar{h}_{x2}$. Iterate the process on $\bar{h}_{xi}$. Finally we get the ultimate generated act $\bar{h}_{xN}$ using some $p_1, ...p_n$ in this process, and $\bar{h}_x^* = \bar{h}_{xN}$. Consider $q_1 \in \mathcal{F}_P$ such that $q_1 \sim \delta_y$ and $\text{supp}(\bar{h}) \cap \text{supp}(q_1) = \emptyset$. Conduct the same process on $\bar{h}_{\omega_1 y q_1}$. In the end we get the ultimate generated act $\bar{h}_{yN}$; note that $\bar{h}_y^* = \bar{h}_{yN}$. By iteratively applying Axiom 3.2 in the process of generating $\bar{h}_{xN}$ and $\bar{h}_{yN}$, we have $\bar{h}_x^* \sim \bar{h}_y^*$. Therefore, $\frac{1}{2}f_x^* + \frac{1}{2}h \sim \frac{1}{2}h_y^* + \frac{1}{2}f$. Since $f_{x_z}^* \propto h_z$ for any $z \in \text{supp}(h)$ and $f \propto h_{y_z}^*$ for any $z \in \text{supp}(f)$, using lemma 2, we get $U(f_x^*) - U(f) = U(h_y^*) - U(h)$. Consider any $h' \in \mathcal{F}_2(f_0,x)$ such that $\text{supp}(h') = \{y,z'\}$, $h'_y = h_y$, and $h'_{z'} = h_z$. So

$$U((f_0^*)_x) - U(f_0) = U(f_x^*) - U(f) = U(h_y^*) - U(h) = U(h_y'^*) - U(h')$$

where the first and last equalities are due to statement 1.

□

Claim 5. For any $f \in \mathcal{F}$ and $f_I^* \in I^*(f)$, $\sum_x \bar{v}(f_x) = U(f_I^*) - U(f)$.

Proof. For any $f \in \mathcal{F}$ and $f_I^* \in I^*(f)$, record $\{x_1, ..., x_M\} \subset X$ and $f^1, f^2, ..., f^M$ from the definition (construction) of f_I^*. Let $f^0 := f$ and $f_{x_1}^* := f^1$. If $M > 1$, for every $i > 1$, pick a $f_{x_i}^* \in F^*(f, x_i)$ such that $(f_{x_i}^*, f^i)$ involve common splits (Subsection A.1). By definition, $\frac{1}{2}f + \frac{1}{2}f^i = \frac{1}{2}f_{x_i}^* + \frac{1}{2}f^{i-1}$. For any $i \in \{1, ..., M\}$, since $f_y \propto f_y^i$ for any $y \in \text{supp}(f)$ and $(f_{x_i}^*)_y \propto f_y^{i-1}$ for any $y \in \text{supp}(f)$, lemma 2

gives the first and last equalities in

$$\frac{1}{2}U\left(f\right)+\frac{1}{2}U\left(f^{i}\right)=U\left(\frac{1}{2}f+\frac{1}{2}f^{i}\right)=U\left(\frac{1}{2}f_{x_i}^{*}+\frac{1}{2}f^{i-1}\right)=\frac{1}{2}U\left(f_{x_i}^{*}\right)+\frac{1}{2}U\left(f^{i-1}\right).$$

Rearranging gives $U\left(f_{x_i}^{*}\right)-U\left(f\right)=U\left(f^{i}\right)-U\left(f^{i-1}\right)$ for all $i\in\{1,...,M\}$. Finally,

$$\begin{aligned}\sum_{x\in\text{supp}(f)}\bar{v}\left(f_x\right)&=\sum_{x\in\text{supp}(f)}U\left(f_x^{*}\right)-U(f)\\&=\sum_{i=1,...,M}U\left(f^{i}\right)-U\left(f^{i-1}\right)=U\left(f^{M}\right)-U\left(f^{0}\right)\\&=U\left(f_I^{*}\right)-U\left(f\right)\end{aligned}$$

as desired. □

Claim 6. For any $f_x\in[0,1]^N$, if $kf_x\in[0,1]^N$ for $k\geq 0$, then $\bar{v}\left(kf_x\right)=k\bar{v}\left(f_x\right)$. Also, $\bar{v}\left(\cdot\right)\geq 0$.

Proof. To show that $\bar{v}$ is homogeneous of degree 1, we focus on binary state space ($N=2$) for ease of exposition. The proof works for any finite state space. Fix $\left(a,b\right),\left(ka,kb\right)\in[0,1]^N$ where $k\geq 0$. Consider the following acts:

f	x	y	z
ω_1	a	$1-a$	0
ω_2	b	0	$1-b$

g	x	y	z
ω_1	ka	$1-ka$	0
ω_2	kb	0	$1-kb$

h	y	z
ω_1	1	0
ω_2	0	1

Note that for any $i\in\text{supp}\left(f\right)$, we have $f_i\propto h_i$. Note also that $g=kf+\left(1-k\right)h$ and $g_x^{*}=kf_x^{*}+\left(1-k\right)h$. Then by lemma 2, $\bar{v}\left(g_x\right)=U\left(g_x^{*}\right)-U\left(g\right)=U\left(kf_x^{*}+\left(1-k\right)h\right)-U\left(g\right)=kU\left(f_x^{*}\right)+\left(1-k\right)U\left(h\right)-U\left(kf+\left(1-k\right)h\right)=k\bar{v}\left(f_x\right)$. So $\bar{v}\left(kf_x\right)=k\bar{v}\left(f_x\right)$ as desired.

To show that $\bar{v}\left(\cdot\right)\geq 0$, we need to show $f_y^{*}\succsim f$ for all $f\in\mathcal{F}$ and $f_y^{*}\in F^{*}\left(f,y\right)$. Suppose $y\notin\text{supp}\left(f\right)$, then it is already established that $\bar{v}\left(f_y\right)=0$. Suppose from now on $y\in\text{supp}\left(f\right)$. We focus on binary state space ($N=2$) for ease of exposition. The proof works for any finite state space. Suppose $ax+\left(1-a\right)z\sim a'x'+\left(1-a'\right)z'\sim a''x''+\left(1-a''\right)z''\sim y$ for distinct x,z,x',z',x'',z'' and $\{x,z,x',z',x'',z''\}\cap\text{supp}(f)=\emptyset$. Pick any $g_1\in F^{*}\left(f,y\right)$ where the split uses $ax+\left(1-a\right)z$ to replace y under ω_1 and uses $a'x'+\left(1-a'\right)z'$ replaces y under ω_2. Let $g_2\in F^{*}\left(f,y\right)$ where the split uses $a'x'+\left(1-a'\right)z'$ to replace y under ω_1 and uses

$a''x'' + (1-a'')\,z''$ to replace y under ω_2. Then by repeatedly applying Axiom 3.2, we have $g_1 \sim g_2$. Let $g_3 \in F^*(f,y)$ where the split uses $a'x' + (1-a')\,z'$ to replace y under ω_1 and uses $ax + (1-a)\,z$ to replace y under ω_2. Then by Axiom 3.2, $g_2 \sim g_3$, then $g_1 \sim g_3$. By Axiom 3.2, $\frac{1}{2}g_1 + \frac{1}{2}g_3 \sim f$. Axiom 3.3 implies $g_1 \succsim \frac{1}{2}g_1 + \frac{1}{2}g_3 \sim f$. Note that claim 2 gives $U\left(f_y^*\right) = U(g_1)$, which gives $f_y^* \sim g_1$, and so $f_y^* \succsim f$ by weak order.

□

Claim 7. $\bar{v}$ is concave.

Proof. Fix any $r, s \in [0,1]^N$, we need to show $\bar{v}(ar + (1-a)\,s) \geq a\bar{v}(r) + (1-a)\,\bar{v}(s)$. Pick $f, g \in \mathcal{F}$ such that $f_x = r$, $g_x = s$, and $\text{supp}(f) \cap \text{supp}(g) = \{x\}$. Let $h \coloneqq af + (1-a)\,g$; clearly,

$$h_x = af_x + (1-a)\,g_x. \tag{A.1}$$

Pick $f_I^* \in I^*(f)$ and $g_I^* \in I^*(g)$ such that $\text{supp}(f_I^*) \cap \text{supp}(g_I^*) = \emptyset$, so lemma 2 gives the second equality in

$$U(h_I^*) = U\left(af_I^* + (1-a)\,g_I^*\right) = aU(f_I^*) + (1-a)\,U(g_I^*), \tag{A.2}$$

where $h_I^* \in I^*(h)$ and the first equality is due to $h_I^* = af_I^* + (1-a)\,g_I^*$. Observe that

$$\begin{aligned}
U(af + (1-a)\,g) &\leq aU(f) + (1-a)\,U(g) \\
[U(h_I^*)] - [U(af + (1-a)\,g)] &\geq [aU(f_I^*) + (1-a)\,U(g_I^*)] - [aU(f) + (1-a)\,U(g)] \\
U(h_I^*) - U(h) &\geq a\,[U(f_I^*) - U(f)] + (1-a)\,[U(g_I^*) - U(g)] \\
\sum_{z \in \text{supp}(h)} \bar{v}(h_z) &\geq a \sum_{z \in \text{supp}(f)} \bar{v}(f_z) + (1-a) \sum_{z' \in \text{supp}(g)} \bar{v}(g_{z'}) \qquad \text{(A.3)} \\
\bar{v}(h_x) &\geq a\bar{v}(f_x) + (1-a)\,\bar{v}(g_x), \qquad \text{(A.4)}
\end{aligned}$$

where the first line is given by Axiom 3.3; the second line is by multiplying (-1) and adding the LHS and RHS of Equation A.2; the third line replaces $h = af + (1-a)\,g$ and rearranges; the fourth line is by claim 5. To arrive at the last line, observe that

for all $z \in \text{supp}(h) \setminus \{x\}$, either $z \in \text{supp}(f)$ or $\text{supp}(g)$ (and not both), so

$$\bar{v}(h_z) = \bar{v}(af_z + (1-a)g_z) = \begin{cases} \bar{v}(af_z) = a\bar{v}(f_z) & \text{if } z \in \text{supp}(f) \\ \bar{v}((1-a)g_z) = (1-a)\bar{v}(g_z) & \text{if } z \in \text{supp}(g) \end{cases},$$

where claim 6 is used, resulting in cancellations on both sides of Equation A.3 that yield the last line. So Equation A.1 implies Equation A.4, as desired. □

Lemma 3. *There exist* $v : \mathcal{F} \times \Omega \to \mathbb{R} \cup \{-\infty\}$ *and* $\tilde{A} \subset \mathcal{F}$ *such that for any* $r \in [0,1]^N$,

$$\bar{v}(r) = \sum_{\omega \in \Omega} r(\omega) \left[\sup_{g \in \tilde{A}} v(g, \omega)\right] - \sup_{g \in \tilde{A}} \left[\sum_{\omega \in \Omega} r(\omega) v(g, \omega)\right].$$

Proof. We have $\bar{v}(\cdot) \geq 0$ due to claim 6and concave due to claim 7. By claim 6, for any $r \in [0,1]^N$, we have $\bar{v}(r) = \left[\sum_{\omega \in \Omega} r(\omega)\right] \bar{v}(\tilde{r})$, where $\tilde{r}(\omega) = \frac{r(\omega)}{\sum_{\omega' \in \Omega} r(\omega')}$. Therefore, we can define $\tilde{v} : \Delta(\Omega) \to \mathbb{R}$ such that $\tilde{v}(\tilde{r}) = \bar{v}(\tilde{r})$ for any $\tilde{r} \in \Delta(\Omega)$, then $\tilde{v}$ is concave and $\tilde{v} \geq 0$. By the definition of $\bar{v}$, $\bar{v}(q) = \tilde{v}(q) = 0$ for degenerate $q \in \Delta(\Omega)$, i.e., there exists one $\omega \in \Omega$ such that $q(\omega) = 1$. By Proposition 15 in Annie Liang's lecture notes entitled *Information and Learning in Economic Theory* (which references Frankel and Kamenica (2019)), there exist a set $\tilde{A} \subset \mathcal{F}$ and a function $v : \tilde{A} \times \Omega \to \mathbb{R} \cup \{-\infty\}$ such that $\tilde{v}(q) = \sum_{\omega \in \Omega} q(\omega) \left[\sup_{g \in \tilde{A}} v(g, \omega)\right] - \sup_{g \in \tilde{A}} \left[\sum_{\omega \in \Omega} q(\omega) v(g, \omega)\right]$. Extend v's domain to $\mathcal{F}$ by setting $v(f, \omega) = 0$ for any $f \in \mathcal{F} \setminus \tilde{A}$ and any $\omega \in \Omega$. Therefore,

$$\begin{aligned} \bar{v}(r) &= \left[\sum_{\omega \in \Omega} r(\omega)\right] \tilde{v}(\tilde{r}) \\ &= \sum_{\omega \in \Omega} r(\omega) \left[\sup_{g \in \tilde{A}} v(g, \omega)\right] - \sup_{g \in \tilde{A}} \left[\sum_{\omega \in \Omega} r(\omega) v(g, \omega)\right]. \end{aligned}$$

□

Step 3: Construction of $\tilde{U}_I$ and U_C from U

For all $f \in \mathcal{F}$, define

$$\tilde{U}_I(f) \coloneqq \sum_{x \in \text{supp}(f)} \sup_{\tilde{g} \in \tilde{A}} \sum_{\omega \in \Omega} f(x \mid \omega)\, v(\tilde{g}, \omega) \tag{A.5}$$

$$U_C(f) \coloneqq U(f) - \tilde{U}_I(f). \tag{A.6}$$

Let

$$\gamma_1 \coloneqq \max_{\tilde{g} \in \tilde{A}} \sum_{\omega \in \Omega} v(\tilde{g}, \omega) \quad \& \quad \gamma_2 \coloneqq \sum_{\omega \in \Omega} \max_{\tilde{g} \in \tilde{A}} v(\tilde{g}, \omega).$$

Claim 8. For any $p \in \mathcal{F}_P$, $\tilde{U}_I(p) = \gamma_1$. For any $I \in \mathcal{F}_I$, $\tilde{U}_I(f) = \gamma_2$.

Proof. For any constant act $p \in \mathcal{F}$, $\tilde{U}_I(p) = \sum_{x \in \text{supp}(f)} \max_{\tilde{g} \in \tilde{A}} \sum_{\omega \in \Omega} p(x)\,[v(\tilde{g}, \omega)] = \sum_{x \in \text{supp}(f)} p(x) \max_{\tilde{g} \in \tilde{A}} \sum_{\omega \in \Omega} [v(\tilde{g}, \omega)] = 1 \cdot \gamma_1$. For any informative act $I \in \mathcal{F}_I$, for each $\omega \in \Omega$, denote by $X_\omega \subset X$ the set of prizes that occur with positive probability under ω. Then $\tilde{U}_I(I) = \sum_{x \in \text{supp}(f)} \max_{\tilde{g} \in \tilde{A}} \sum_{\omega \in \Omega} I(x|\omega)\,[v(\tilde{g}, \omega)] = \sum_{\omega \in \Omega} \sum_{x \in X_\omega} \max_{\tilde{g} \in \tilde{A}} I(x|\omega)\,[v(\tilde{g}, \omega)] = \sum_{\omega \in \Omega} \sum_{x \in X_\omega} I(x|\omega) \max_{\tilde{g} \in \tilde{A}} [v(\tilde{g}, \omega)] = \sum_{\omega \in \Omega} 1 \cdot \max_{\tilde{g} \in \tilde{A}} [v(\tilde{g}, \omega)] = \gamma_2$. □

Claim 9. For any $p, q \in \mathcal{F}_P$, $p \sim q$ implies $U_C(p) = U_C(q)$. For any $f, g \in \mathcal{F}_I$, $f \sim g$ implies $U_C(f) = U_C(g)$.

Proof. The immediate consequences of lemma 2, claim 8, and Equation A.6. □

Step 4: State-dependent utility $\bar{u}$

Pick N distinct prizes to form an enumerated set of benchmark prizes $B \coloneqq \{b_1, ..., b_N\} \subset X$. Let $\mathcal{F}_B \coloneqq \{f \in \mathcal{F} : \text{supp}(f) \cap B = \emptyset\}$. Consider the *benchmark act* $f_0 \in \mathcal{F}$ such that $f_0(b_i \mid \omega_i) = 1$ for all $i \in \{1, ..., N\}$. For any $x \in X \setminus B$ and $\omega_i \in \Omega$, denote by f_{x,ω_i} the *statewise simple act* satisfying $f(x \mid \omega_i) = 1$ and $f(b_j \mid \omega_j) = 1$ for all $j \in \{1, ..., N\} \setminus \{i\}$. For any $x \in X \setminus B$ and $\omega_i \in \Omega$, let

$$\bar{u}(x, \omega_i) \coloneqq U(f_{x,\omega_i}) - \frac{N-1}{N} U(f_0) - \frac{1}{N} \gamma_2. \tag{A.7}$$

Claim 10. For any simple act $f^s \in \mathcal{F}_S \cap \mathcal{F}_B$, $U_C(f^s) = \sum_{\omega \in \Omega} \sum_{z \in \text{supp}(f)} f^s(z \mid \omega)\, \bar{u}(z, \omega)$.

Proof. Take a qualifying f^s and denote the prize it yields in state ω_i (with probability 1) by x_i. Observe for later that $\bar{u}(x_i,\omega_i) = 1 \cdot \bar{u}(x_i,\omega_i) = f^s(x_i \mid \omega_i)\,\bar{u}(x_i,\omega_i) = \sum_{z\in\text{supp}(f)} f^s(z \mid \omega_i)\,\bar{u}(z,\omega_i)$ for all i. Observe that $\frac{1}{N}f^s + \frac{N-1}{N}f_0 = \oplus_{i=\{1,\ldots,N\}}\frac{1}{N}f_{x_i,\omega_i}$. Because $f^s \propto f_0$ and $\text{supp}(f_{x_i,\omega_i}) \cap \text{supp}(f_{x_j,\omega_j}) = \emptyset$ for all distinct i,j, lemma 2 gives

$$\frac{1}{N}U(f^s) + \frac{N-1}{N}U(f_0) = \frac{1}{N}\sum_{i\in\{1,\ldots,N\}} U(f_{x_i,\omega_i}). \tag{A.8}$$

Equation A.7 gives $\sum_{i=\{1,\ldots,N\}} U(f_{x_i,\omega_i}) = \sum_{i=\{1,\ldots,N\}} \bar{u}(x_i,\omega_i) + (N-1)\,U(f_0) + \gamma_2$, which can be used to simplify Equation A.8 into $U(f^s) - \tilde{U}_I(f^s) = \sum_{i\in\{1,\ldots,N\}} \bar{u}(x_i,\omega_i) = \sum_{\omega\in\Omega}\sum_{z\in\text{supp}(f)} f^s(z \mid \omega)\,\bar{u}(z,\omega)$, where $\tilde{U}_I(f^s) = \gamma_2$ due to claim 8 and the second equality is due to an earlier observation in the proof. Since $U_C(f^s) = U(f^s) - \tilde{U}_I(f^s)$ (by Equation A.6), we are done. □

Claim 11. For any informative act $I \in \mathcal{F}_I \cap \mathcal{F}_B$, $U_C(I) = \sum_{\omega\in\Omega}\sum_{z\in\text{supp}(I)} I(z \mid \omega)\,\bar{u}(z,\omega)$.

Proof. For any informative act $I \in \mathcal{F}_I \cap \mathcal{F}_B$, define its corresponding simple act set

$$S(I) := \{f^s \in \mathcal{F}_S \cap \mathcal{F}_B : f^s(\omega_i) \in \text{supp}(f(\omega_i)), \forall i \in \{1,\ldots,N\}\}.$$

Observe that I is a convex combination of the acts in $S(I)$, i.e., $I = \oplus_{f\in S(I)} a_f f$ where $a_f \in [0,1]$ for all $f \in S(I)$ and $\sum_{f\in S(I)} a_f = 1$. Since $f \propto g$ for any $f,g \in S(I)$, by lemma 2, we have $U(I) = \sum_{f\in S(I)} a_f U(f)$. claim 8 gives $\tilde{U}_I(f) = \tilde{U}_I(I)$ for all $f \in S(I)$, so $U_C(I) = \sum_{f\in S(I)} a_f U_C(f)$. Finally,

$$\begin{aligned} U_C(I) &= \sum_{f\in S(I)} a_f U_C(f) = \sum_{f\in S(I)}\sum_{z\in\text{supp}(f)}\sum_{\omega\in\Omega} a_f f(z \mid \omega)\,\bar{u}(z,\omega) \\ &= \sum_{z\in\text{supp}(I)}\sum_{\omega\in\Omega} I(z \mid \omega)\,\bar{u}(z,\omega), \end{aligned}$$

where the second equality is from claim 10 and the third equality is due to $I(x \mid \omega) = \sum_{f\in S(I)} a_f f(x \mid \omega)$ for any $x \in \text{supp}(f)$ and $\omega \in \Omega$. □

Step 5: State-independent utility u

Claim 12. There exists $u : X \to R$ such that $U_C(p) = \sum_{z\in\text{supp}(p)} u(z)\,p(z)$ for all constant act $p \in \mathcal{F}_P$.

Proof. By the definition of $\tilde{U}_I$, $\tilde{U}_I(p) = \sum_{\omega\in\Omega}\left[\max_{\tilde{g}\in\tilde{A}} u(\tilde{g},\omega)\right]$ for all $p \in \mathcal{F}_P$. By lemma 2, U is affine on the set of all constant acts $\mathcal{F}_P$, then $U_C(p) = U(p) - \tilde{U}_I(p)$ is affine on $\mathcal{F}_P$. It is well-known that there exists $u : X \to R$ such that $U_C(p) = \sum_{z\in\text{supp}(p)} u(z)\, p(z)$ for all $p \in \mathcal{F}_P$. Note that $u(x) = U_C(\delta_x)$ for all $x \in X$. □

Claim 13. For any $\omega \in \Omega$, there exist $a_\omega \geq 0$ and $\eta_\omega \in \mathbb{R}$ such that $\bar{u}(z,\omega) = a_\omega u(z) + \eta_\omega$ for all $z \in X \setminus B$.

Proof. Fix any $\omega \in \Omega$. Fix any $z \in X \setminus B$. Suppose $\delta_x \succ \delta_z \succ \delta_y$ for some $x, y \in X\setminus B$. By Continuity in Axiom 3, there exists $a \in (0,1)$ such that $a\delta_x + (1-a)\delta_y \sim \delta_z$, let $q := a\delta_x + (1-a)\delta_y$. Fix a simple act $f^s \in \mathcal{F}_S \cap \mathcal{F}_B$ such that $f^s(\omega) = z$ and $\{\delta_x, \delta_y\} \cap \text{supp}(f^s) = \emptyset$. The existence of f^s is guaranteed by $|X\setminus B| = \infty$ and $|\Omega| < \infty$. and Axiom 3.2 gives $f^s z q \sim f^s$, then lemma 2 gives $U(f^s z q) = U(f^s)$. Consider $f_1 \in \mathcal{F}_S \cap \mathcal{F}_B$ where $f_1(\omega) = x$ and $f_1(\omega') = f^s(\omega')$ for all $\omega' \in \Omega\setminus\{\omega\}$. Consider $f_2 \in \mathcal{F}_\mathcal{S} \cap \mathcal{F}_B$ where $f_2(\omega) = y$ and $f_2(\omega') = f^s(\omega')$ for all $\omega' \in \Omega\setminus\{\omega\}$. Then $f_1 \propto f_2$ and $f^s z q = a f_1 + (1-a) f_2$. lemma 2 gives $aU(f_1) + (1-a)U(f_2) = U(f^s z q) = U(f^s)$. Due to $\tilde{U}_I(f^s) = \tilde{U}_I(f_1) = \tilde{U}_I(f_2)$ (claim 8), we have $aU_C(f_1) + (1-a)U_C(f_2) = U_C(f^s)$. By claim 10, after simplification, we obtain

$$a\bar{u}(x,\omega) + (1-a)\bar{u}(y,\omega) = \bar{u}(z,\omega). \tag{A.9}$$

Since $q \sim z$, claim 9 gives $U_C(q) = U_C(z)$. claim 12 then gives

$$au(x) + (1-a)u(y) = u(z). \tag{A.10}$$

Therefore, Equation A.9 and Equation A.10 hold for any $x, y, z \in X \setminus B$ such that $\delta_y \succ \delta_z \succ \delta_x$ (with (x,y,z)-dependent a). If $\bar{u}(x',\omega) = \bar{u}(y',\omega)$ for all $x', y' \in X\setminus B$, then let $a_\omega = 0$ and find $\eta_\omega \in \mathbb{R}$ such that $\bar{u}(z',\omega) = a_\omega u(z') + \eta_\omega$ for all $z' \in X \setminus B$. If $\bar{u}(x',\omega) \neq \bar{u}(y',\omega)$ for some $x', y' \in X\setminus B$, then there exist $a_\omega > 0$ and $\eta_\omega \in \mathbb{R}$ such that $\bar{u}(z',\omega) = a_\omega u(z') + \eta_\omega$ for all $z' \in X \setminus B$. □

Claim 14. $a_\omega > 0$ for all $\omega \in \Omega$.

Proof. Fix any $\omega \in \Omega$. In the last part of proof of claim 13, $a_\omega = 0$ only if $\bar{u}(x',\omega) = \bar{u}(y',\omega)$ for all $x', y' \in X\setminus B$. Take any distinct $x', y' \in X\setminus B$. From the definition of $\bar{u}$ (Equation A.7), $\bar{u}(x',\omega) = \bar{u}(y',\omega)$ only if $U(f_{x',\omega}) = U(f_{y',\omega})$. By lemma 2, this means $f_{x',\omega} \sim f_{y',\omega}$. Suppose without loss of generality that $x' > y'$,

then Monotonicity in Axiom 3 gives $x' \succ y'$, then Axiom 3.2 gives $f_{x',\omega} \succ f_{y',\omega}$, a contradiction. □

Claim 15. If $\{a_\omega\}_{\omega\in\Omega}$ and $\{\eta_\omega\}_{\omega\in\Omega}$ satisfy $\bar{u}(z,\omega) = a_\omega u(z) + \eta_\omega$ for all $z \in X\backslash B$ and $\omega \in \Omega$, then $\sum_\omega a_\omega = 1$ and $\sum_\omega \eta_\omega = 0$.

Proof. For any degenerate constant act $x \in \mathcal{F}_X$, consider the split $x_I^* \in \mathcal{F}_I \cap \mathcal{F}_B$ such that for each $\omega_i \in \Omega$, x is replaced by $q_i \in \mathcal{F}_P \cap \mathcal{F}_B$ where $q_i \sim x$, and $\text{supp}(q_j) \cap \text{supp}(q_k) = \emptyset$ for any distinct j, k. The existence of x_I^* is guaranteed by the space X, Monotonicity (in Axiom 3), and Continuity (in Axiom 3). Note that claim 9 gives $U_C(q_i) = U_C(\delta_x)$ for all q_i. By claim 10,

$$U_C(q_i) = \sum_{z\in\text{supp}(q_i)} q_i(z)\, u(z) = u(x) = U_C(\delta_x).$$

Meanwhile, since x_I^* is an informative act, using claim 11, we have

$$\begin{aligned} U_C(x_I^*) &= \sum_{\omega_i\in\Omega}\sum_{z\in\text{supp}(p_I)} x_I^*(z \mid \omega_i)\,\bar{u}(z,\omega_i) = \sum_{\omega_i\in\Omega}\sum_{z\in\text{supp}(q_i)} q_i(z)\,\bar{u}(z,\omega_i) \\ &= \sum_{\omega_i\in\Omega}\sum_{z\in\text{supp}(q_i)} q_i(z)\,(a_{\omega_i}u(z) + \eta_{\omega_i}) = \sum_{\omega_i\in\Omega}(a_{\omega_i}u(x) + \eta_{\omega_i}) \\ &= u(x)\cdot\sum_{\omega_i\in\Omega} a_{\omega_i} + \sum_{\omega_i\in\Omega}\eta_{\omega_i}. \end{aligned}$$

Moreover, from claim 5 and lemma 3,

$$\begin{aligned} U(x_I^*) - U(\delta_x) &= \sum_{\omega\in\Omega}\left[\max_{\tilde{g}\in\tilde{A}} v(\tilde{g},\omega)\right] - \max_{\tilde{g}\in\tilde{A}}\left[\sum_{\omega\in\Omega} v(\tilde{g},\omega)\right] \\ U(x_I^*) - \sum_{\omega\in\Omega}\left[\max_{\tilde{g}\in\tilde{A}} v(\tilde{g},\omega)\right] &= U(\delta_x) - \max_{\tilde{g}\in\tilde{A}}\left[\sum_{\omega\in\Omega} v(\tilde{g},\omega)\right] \\ U(x_I^*) - \tilde{U}_I(x_I^*) &= U(\delta_x) - \tilde{U}_I(\delta_x) \\ U_C(x_I^*) &= U_C(\delta_x), \end{aligned}$$

where the third equality uses claim 8 and the fourth equality is by definition (Equa-

tion A.6). Therefore,

$$u(x) \cdot \sum_{\omega_i \in \Omega} a_{\omega_i} + \sum_{\omega_i \in \Omega} \eta_{\omega_i} = U_C(x_I^*) = U_C(\delta_x) = u(x),$$

and this holds for any $x \in X$. By Monotonicity in Axiom 3, there exist $x, y \in X$ such that $u(x) \neq u(y)$. So $\sum_\omega a_\omega = 1$ and $\sum_\omega \eta_\omega = 0$. □

Let

$$\mu(\omega) = a_\omega \tag{A.11}$$

for all $\omega \in \Omega$. Then for any $x \in X \setminus B$, $u(x) = u(x) \cdot \sum_{\omega_i \in \Omega} a_{\omega_i} + \sum_{\omega_i \in \Omega} \eta_{\omega_i} = \sum_{\omega \in \Omega} \bar{u}(x, \omega)$.

Step 6: Demonstrating that U_C is SEU

Definition 6. Given any $\psi \in \Delta(\Omega)$ and $\phi : X \to \mathbb{R}$, define the function $SEU_{\psi,\phi} : \mathcal{F} \to \mathbb{R}$ by

$$SEU_{\psi,\phi}(f) := \sum_{\omega \in \Omega} \psi(\omega) \sum_{x \in \text{supp}(f)} f(x \mid \omega) \phi(x).$$

We want to show $U_C = SEU_{\mu,u}$ (recall that U_C is defined in Equation A.6, u is from claim 12, and μ is from Equation A.11).

Claim 16. For any informative act $I \in \mathcal{F}_I$, $U_C(I) = SEU_{\mu,u}(I)$.

Proof. For any informative act $I \in \mathcal{F}_B$, by claim 11 and claim 15,

$$\begin{aligned} U_C(I) &= \sum_{z \in \text{supp}(I)} \sum_{\omega \in \Omega} I(z \mid \omega) \bar{u}(z, \omega) = \sum_{z \in \text{supp}(I)} \sum_{\omega \in \Omega} I(z \mid \omega) (\mu(\omega) u(z) + \eta_\omega) \\ &= \sum_{\omega \in \Omega} \mu(\omega) \sum_{z \in \text{supp}(I)} I(z \mid \omega) u(z). \end{aligned}$$

For any simple act $f^s \in \mathcal{F}_S \setminus \mathcal{F}_B$, pick $q_i \in \Delta(X \setminus B)$ for each $i \in \{1, ..., N\}$ such that $q_i \sim f^s(\omega_i)$, $\text{supp}(q_i) \cap \text{supp}(f^s) = \emptyset$, and $\text{supp}(q_i) \cap \text{supp}(q_j) = \emptyset$ for distinct i, j. From $q_i \sim f^s(\omega_i)$, claim 9 gives $U_C(q_i) = U_C(f^s(\omega_i))$, thus $\sum_{x \in \text{supp}(q_i)} q_i(x) u(x) = u(f^s(\omega_i))$. Iteratively use $q_i \in \Delta(X \setminus B)$ to replace $f^s(\omega_i)$ for each $\omega_i \in \Omega$. Denote the resulting act by f^*, note that $f^* \in \mathcal{F}_\mathcal{I} \cap \mathcal{F}_B$. By multiple uses of Axiom 3.2,

$f^* \sim f^s$, so claim 9 gives $U_C(f^*) = U_C(f^s)$. Then,

$$\begin{aligned} U_C(f^*) = U_C(f^s) &= \sum_{\omega\in\Omega} \mu(\omega) \sum_{z\in \text{supp}(I)} I(z \mid \omega)\, u(z) \\ &= \sum_{\omega\in\Omega} \mu(\omega) \sum_{x\in \text{supp}(q_i)} q_i(x)\, u(x) = \sum_{\omega\in\Omega} \mu(\omega)\, u\left(f^s(\omega_i)\right). \end{aligned}$$

Since every informative act $I \in \mathcal{F}_I \setminus \mathcal{F}_B$ is equal to a convex combination of the simple acts in $S(I)$, then by the same argument in claim 11, we have $U_C(I) = \sum_{\omega\in\Omega} \mu(\omega) \sum_{z\in\text{supp}(I)} I(z \mid \omega)\, u(z)$. □

Lemma 4. *For any act $f \in \mathcal{F}$, $U_C(f) = SEU_{\mu,u}(f)$.*

Proof. claim 12 has proved the case for constant acts and claim 16 has proved the case for informative acts. Fix any other $f \in \mathcal{F}$, generate a $f_I^* \in I^*(f)$ and denote the lottery replacement of $z \in \text{supp}(f(\omega_i))$ under $\omega_i \in \Omega$ by $q_i^z \in \Delta(X)$. For all $i \in \{1, ..., N\}$ and $z \in \text{supp}(f(\omega_i))$, recall from the definition of f_I^* that $q_i^z \sim z$, so claim 9 gives $U_C(q_i^z) = U_C(\delta_z)$, and claim 12 further gives $\sum_{x\in\text{supp}(q_i)} q_i^z(x)\, u(x) = u(z)$. Therefore,

$$\begin{aligned} U_C(f) = U_C(f_I^*) &= \sum_{\omega\in\Omega} \mu(\omega) \sum_{z\in\text{supp}\left(f_I^*\right)} f_I^*(z \mid \omega)\, u(z) \\ &= \sum_{\omega\in\Omega} \mu(\omega) \sum_{z\in\text{supp}(f)} f(z \mid \omega) \left[\sum_{x\in\text{supp}(q_i)} q_i^z(x)\, u(x)\right] \\ &= \sum_{\omega\in\Omega} \mu(\omega) \sum_{z\in\text{supp}(f)} f(z \mid \omega)\, u(z) = SEU_{\mu,u}(f). \end{aligned}$$

where the first equality is obtained from rearranging claim 5 for $U(f) - \tilde{U}_I(f) = U(f_I^*) - \tilde{U}_I(f_I^*)$ and using the definition of U_C (Equation A.6); the second equality is due to $f_I^* \in \mathcal{F}_I$ and claim 16; the third equality is by the definition of f_I^*; and the fourth equality is established earlier in the proof. □

Step 7: Deriving V

Recall that μ has full support by claim 14, so dividing by $\mu(\omega)$ is well-defined. From lemma 3 we get

$$\begin{aligned}
\tilde{U}_I(f) &= \sum_{x\in\text{supp}(f)} \sup_{\tilde{g}\in\tilde{A}} \sum_{\omega\in\Omega} f(x\mid\omega)\, v(\tilde{g},\omega) \\
&= \sum_{x\in\text{supp}(f)} \sup_{\tilde{g}\in\tilde{A}} \sum_{\omega\in\Omega} \left[\frac{\sum_{\omega'\in\Omega}\mu(\omega')\, f(x\mid\omega')}{\sum_{\omega'\in\Omega}\mu(\omega')\, f(x\mid\omega')}\frac{\mu(\omega)}{\mu(\omega)}\right] f(x\mid\omega)\, v(\tilde{g},\omega) \\
&= \sum_{x\in\text{supp}(f)} \sum_{\omega'\in\Omega} \mu(\omega')\, f(x\mid\omega') \sup_{\tilde{g}\in\tilde{A}} \left[\sum_{\omega\in\Omega} \frac{\mu(\omega)\, f(x\mid\omega)}{\sum_{\omega'\in\Omega}\mu(\omega')\, f(x\mid\omega')}\frac{v(\tilde{g},\omega)}{\mu(\omega)}\right] \\
&= \sum_{x\in\text{supp}(f)} \sum_{\omega\in\Omega} \mu(\omega)\, f(x\mid\omega) \sup_{\tilde{g}\in\tilde{A}} \left[\sum_{\omega\in\Omega} \mu_{x,f}(\omega)\frac{v(\tilde{g},\omega)}{\mu(\omega)}\right]. \qquad \text{(A.12)}
\end{aligned}$$

Let

$$V(\mu_{x,f}) := \sup_{\tilde{g}\in\tilde{A}} \left[\sum_{\omega\in\Omega} \mu_{x,f}(\omega)\frac{v(\tilde{g},\omega)}{\mu(\omega)}\right]. \qquad \text{(A.13)}$$

Finally, because $U(f) = U_C(f) + \tilde{U}_I(f)$ by definition (Equation A.6), we have

$$U(f) = U_C(f) + \sum_{x\in\text{supp}(f)} \sum_{\omega\in\Omega} \mu(\omega)\, f(x\mid\omega)\, V(\mu_{x,f})$$

as desired.

A.2 Proof of Theorem 2

We prove "only if" as the other direction is straightforward after utilizing the fact that an IC representation attains its $\max$, which precludes the case of "vertical hyperplane."

Since $\succsim$ admits an IC-V representation, the proof of theorem 1 can be replayed until Equation A.12. Note that if

$$\max_{\tilde{g}\in\tilde{A}} \left[\sum_{\omega\in\Omega} \mu_{x,f}(\omega)\frac{v(\tilde{g},\omega)}{\mu(\omega)}\right]$$

is undefined (i.e., there is no argmax $\tilde{g}$) for some $\mu_{x,f}\in\Delta(\Omega)$, then $V(\mu_{x,f})$ (Equa-

tion A.13) will have infinite (or negative infinite) slopes at at least one of the degenerate posteriors, which contradicts the fact that V is Lipschitz continuous. This corresponds to the known issue of "vertical" supporting hyperplanes of $V(\cdot)$ towards the boundary. Hence

$$\sup_{\tilde{g}\in\tilde{A}}\left[\sum_{\omega\in\Omega}\mu_{x,f}(\omega)\frac{v(\tilde{g},\omega)}{\mu(\omega)}\right]=\max_{\tilde{g}\in\tilde{A}}\left[\sum_{\omega\in\Omega}\mu_{x,f}(\omega)\frac{v(\tilde{g},\omega)}{\mu(\omega)}\right].$$

Excluding never-optimal $\tilde{g}$ from $\tilde{A}$ (that is, $\tilde{g}$ that is never the argmax for any $\mu_{x,f}\in\Delta(\Omega)$), the previous argument implies that $\max_{\tilde{g}\in\tilde{A}}v(\tilde{g},\omega)$ and $\min_{\tilde{g}\in\tilde{A}}v(\tilde{g},\omega)$ are well-defined (attained) for all $\omega\in\Omega$ (these are the maximum and minimum intercepts of the hyperplanes $v(\tilde{g},\cdot)$). So $u^{+}\coloneqq\max_{\tilde{g}\in\tilde{A},\omega\in\Omega}\frac{v(\tilde{g},\omega)}{\mu(\omega)}$ and $u^{-}\coloneqq\min_{\tilde{g}\in\tilde{A},\omega\in\Omega}\frac{v(\tilde{g},\omega)}{\mu(\omega)}$ are well-defined

Suppose for now that $u^{-}<u^{+}$. Let β satisfy $\beta(u(1)-u(0))=u^{+}-u^{-}$, where u is the Bernoulli utility function already obtained. Observe that $\beta>0$ and $\frac{1}{\beta}\frac{v(\tilde{g},\omega)}{\mu(\omega)}\in[u(0),u(1)]$ for any $\tilde{g}\in\tilde{A}$ and $\omega\in\Omega$. For any $\tilde{g}\in\tilde{A}$ and $\omega\in\Omega$, let (define) $p_{\omega}(\tilde{g})$ satisfy

$$p_{\omega}(\tilde{g})u(1)+(1-p_{\omega}(g))u(0)=\frac{1}{\beta}\frac{v(\tilde{g},\omega)}{\mu(\omega)}.$$

Construct $A\subseteq\mathcal{F}$ by including $g\in\mathcal{F}$ in A if and only if there exists $\tilde{g}\in\tilde{A}$ such that $g(\omega)=p_{\omega}(\tilde{g})\delta_{1}+(1-p_{\omega}(\tilde{g}))\delta_{0}$ for all $\omega\in\Omega$. Continuing from Equation A.12, we now have

$$\begin{aligned}\tilde{U}_{I}(f)&=\sum_{x\in\text{supp}(f)}\sum_{\omega\in\Omega}\mu(\omega)f(x\mid\omega)\max_{\tilde{g}\in\tilde{A}}\left[\sum_{\omega\in\Omega}\mu_{x,f}(\omega)\frac{v(\tilde{g},\omega)}{\mu(\omega)}\right]\\&=\sum_{x\in\text{supp}(f)}\sum_{\omega\in\Omega}\mu(\omega)f(x\mid\omega)\max_{g\in A}\left[\sum_{\omega\in\Omega}\mu_{x,f}(\omega)\beta\left[g(\omega)(1)u_{1}(1)+g(\omega)(0)u_{1}(0)\right]\right]\\&=\beta\sum_{x\in\text{supp}(f)}\sum_{\omega\in\Omega}\mu(\omega)f(x\mid\omega)\left[\max_{g\in A}\sum_{\omega\in\Omega}\mu_{x,f}(\omega)\sum_{y\in\text{supp}(g)}g(y\mid\omega)u(y)\right]\end{aligned}$$

Because $U(f)=U_{C}(f)+\tilde{U}_{I}(f)$ by definition, we now have

$$U(f)=U_{C}(f)+\beta U_{I}(f)$$

where $U_I(f)$ is from Equation 2.2. Let $\alpha > 0$ satisfy $\beta = \frac{1-\alpha}{\alpha}$ and multiply $U(f)$ by α, we now obtain an IC representation with parameters (μ, u, α, A).

Suppose instead that $u^- = u^+$ (this is the special case where information has no value). Then for any $\omega \in \Omega$, we have $\frac{v(\tilde{g},\omega)}{\mu(\omega)} = c \in \mathbb{R}$ for all $\tilde{g} \in \tilde{A}$. So every act produces the same information value and therefore $f \succsim g$ if and only $U_C(f) \geq U_C(g)$. With freedom, we conclude that $\succsim$ has a IC representation with parameters $(\mu, u, \alpha = 1, A = \{\delta_0\})$ (A still needs to be well-specified for the $\max$ operation to be well-defined).

A.3 Proof of Proposition 1

The "if" direction is straightforward noting that for any f,

$$\begin{aligned}
&\sum_{x \in \text{supp}(f)} \sum_{\omega \in \Omega} \mu(\omega) f(x \mid \omega) [\lambda \cdot \mu_{x,f}] \\
&= \sum_{x \in \text{supp}(f)} \sum_{\omega \in \Omega} \mu(\omega) f(x \mid \omega) \left[\sum_{\omega' \in \Omega} \lambda_{\omega'} \frac{\mu(\omega') f(x \mid \omega')}{\sum_{\omega'' \in \Omega} \mu(\omega'') f(x \mid \omega'')} \right] \\
&= \sum_{x \in \text{supp}(f)} \sum_{\omega' \in \Omega} [\lambda_{\omega'} \mu(\omega') f(x \mid \omega')] \\
&= \sum_{\omega' \in \Omega} \sum_{x \in \text{supp}(f)} [\lambda_{\omega'} \mu(\omega') f(x \mid \omega')] \\
&= \sum_{\omega' \in \Omega} [\lambda_{\omega'} \mu(\omega')] \\
&= \lambda \cdot \mu,
\end{aligned}$$

which is independent of f.

We show the "only if" direction. Suppose $\succsim$ admits an IC-V representation (μ_1, u_1, V_1) and it also admits an IC-V representation (μ_2, u_2, V_2). For any $i \in \{1, 2\}$ and $f \in \mathcal{F}$, let $U_I^i(f) = \sum_{x \in \text{supp}(f)} \sum_{\omega \in \Omega} \mu_i(\omega) f(x \mid \omega) V_i(\mu_{x,f})$.

For any $x \in X$,

$$U_i(\delta_x) = SEU_{\mu_i u_i}(\delta_x) + V_i(\mu_i) = SEU_{\mu_i, u_i + V_i(\mu_i)}(\delta_x)$$

for $i \in \{1, 2\}$. Since $\delta_x \succsim \delta_y$ if and only if $U_i(\delta_x) \geq U_i(\delta_y)$ for all $i \in \{1, 2\}$, then by the classical SEU identification result, there exists $\kappa > 0$ and $c' \in \mathbb{R}$ such that

$u_2(x)+V_2(\mu_2) = \kappa u_1(x)+V_1(\mu_1)+c'$ for any $x \in X$. Let $c = V_1(\mu_1)+c'-V_2(\mu_2)$. We have $u_2(x) = \kappa u_1(x)+c$. Therefore, for prize 1 and prize 0, we have $u_2(1)-u_2(0) = \kappa(u_1(1)-u_1(0))$.

We now argue that $\mu_1 = \mu_2$. For any $x \in X$, there exists a unique $a_x \in \mathbb{R}$ such that

$$u_i(x) = a_x u_i(1) + (1-a_x) u_i(0) = a_x(u_i(1)-u_i(0)) + u_i(0) \tag{A.14}$$

for any $i \in \{1,2\}$. Let $N = |\Omega|$. For the following, we abuse notation and write z and δ_z interchangeably.

1. Let $f_1 \in \mathcal{F}$ be such that $f_1(\omega_1) = 5$, $f_1(\omega_2) = 6$ and $f_1(\omega_i) = -1$ for any $\omega_i \in \Omega \setminus \{\omega_1, \omega_2\}$.

2. Let $f_2 \in \mathcal{F}$ be such that $f_2(\omega_1) = 6$, $f_2(\omega_2) = 5$ and $f_2(\omega_i) = -1$ for any $\omega_i \in \Omega \setminus \{\omega_1, \omega_2\}$.

3. Let $g \in \mathcal{F}$ be such that $g(\omega_1) = 3$, $g(\omega_2) = 4$ and $g(\omega_i) = -1$ for any $\omega_i \in \Omega \setminus \{\omega_1, \omega_2\}$.

4. Let $h \in \mathcal{F}$ be such that $h(\omega_1) = 7$, $h(\omega_2) = 8$ and $h(\omega_i) = -1$ for any $\omega_i \in \Omega \setminus \{\omega_1, \omega_2\}$.

There exists $t_1, t_2 \in (0,1)$ such that $f_1 \sim t_1 g + (1-t_1) h$ and $f_2 \sim t_2 g + (1-t_2) h$. Then, for any $i \in \{1,2\}$: $U_i(f_1) = U_i(t_1 g + (1-t_1) h)$ and $U_i(f_1) = t_1 U_i(g) + (1-t_1) U_i(h)$. Because f_1, g, h have equal information value i.e., $U_I^i(f_1) = U_I^i(g) = U_I^i(h)$, $\mu_i(\omega_1) u_i(5) + \mu_i(\omega_2) u_i(6) = t_1(\mu_i(\omega_1) u_i(3) + \mu_i(\omega_2) u(4)) + (1-t_1)(\mu_i(\omega_1) u_i(7) + \mu_i(\omega_2) u_i(8))$, so Equation A.14 gives

$$\mu_i(\omega_1) a_5 + \mu_i(\omega_2) a_6 = t_1(\mu_i(\omega_1) a_3 + \mu_i(\omega_2) a_4) + (1-t_1)(\mu_i(\omega_1) a_7 + \mu_i(\omega_2) a_8).$$

Similarly, because f_2, g, h have equal information value, we obtain

$$\mu_i(\omega_1) a_6 + \mu_i(\omega_2) a_5 = t_2(\mu_i(\omega_1) a_3 + \mu_i(\omega_2) a_4) + (1-t_2)(\mu_i(\omega_1) a_7 + \mu_i(\omega_2) a_8).$$

Notice we obtain the same two equations for two unknowns $\mu_i(\omega_1), \mu_i(\omega_2)$ for each of $i \in \{1,2\}$, therefore we have $\mu_1(\omega_1) = \mu_2(\omega_1)$ and $\mu_1(\omega_2) = \mu_2(\omega_2)$. Re-

peating the same arguments for other ω's gives $\mu_1(\omega) = \mu_2(\omega)$ for all $\omega \in \Omega$, hence $\mu_1 = \mu_2$. We denote the unique prior as μ from now on.

Next, let I^* be a simple act (see Subsection A.1) such that $x \succ 0$ for any $x \in \text{supp}(I^*)$. We argue that $U_I^2(I^*) - U_I^2(\delta_0) = \kappa(U_I^1(I^*) - U_I^1(\delta_0))$. There exists $\epsilon \in (0,1)$ and constant act (lottery) $p \in \mathcal{F}_P$ such that $\epsilon f + (1-\epsilon)\delta_0 \sim p$. Then for any $i \in \{1,2\}$,

$$U^i(\epsilon f + (1-\epsilon)\delta_0) = U^i(p)$$

$$\epsilon U_c^i(I^*) + (1-\epsilon)U_c^i(\delta_0) + \epsilon U_I^i(I^*) + (1-\epsilon)U_I^i(\delta_0) = U_c^i(p) + U_I^i(p)$$

$$\epsilon\left(U_I^i(I^*) - U_I^i(\delta_0)\right) = U_c^i(p) - \epsilon U_c^i(I^*) - (1-\epsilon)U_c^i(\delta_0).$$

The third equality follows from $U_I^i(\delta_0) = U_I^i(p)$. Therefore,

$$\frac{\epsilon(U_I^1(I^*) - U_I^1(\delta_0))}{\epsilon(U_I^2(I^*) - U_I^2(\delta_0))} = \frac{U_c^1(p) - \epsilon U_c^1(I^*) - (1-\epsilon)U_c^1(\delta_0)}{U_c^2(p) - \epsilon U_c^2(I^*) - (1-\epsilon)U_c^2(\delta_0)} = \frac{u_2(1) - u_2(0)}{u_1(1) - u_1(0)} = \kappa$$

Let $\mathcal{F}^* = \{f \in \mathcal{F} : I^* \succ f \succ \delta_0, f \cap I^* = \emptyset, f \cap \delta_0 = \emptyset\}$. Then, for any $f \in \mathcal{F}^*$, we show $U_I^2(f) - \kappa U_I^1(f) = U_I^2(\delta_0) - \kappa U_I^1(\delta_0)$. Since there exists a unique $a_f \in (0,1)$ such that $a_f I^* + (1-a_f)\delta_0 \sim f$. Then, for any $i \in \{1,2\}$:

$$U^i(f) = U(a_f I^* + (1-a_f)\delta_0)$$

$$U_c^i(f) + U_I^i(f) = a_f U_c^i(I^*) + a_f U_I^i(I^*) + (1-a_f)U_c^i(\delta_0) + (1-a_f)U_I^i(\delta_0)$$

$$U_I^i(f) - U_I^i(\delta_0) = a_f\left(U_I^i(I^*) - U_I^i(\delta_0)\right) + a_f U_c^i(I^*) + (1-a_f)U_c^i(\delta_0) - U_c^i(f).$$

Since there exists $k \in \mathbb{R}$ such that $a_f U_c^i(I^*) + (1-a_f)U_c^i(\delta_0) - U_c^i(f) = k(u_i(1) - u_i(0))$ for any $i \in \{1,2\}$, we have $\kappa(a_f U_c^1(I^*) + (1-a_f)U_c^1(\delta_0) - U_c^1(f)) = a_f U_c^1(I^*) + (1-a_f)U_c^1(\delta_0) - U_c^1(f)$. Then, $\kappa U_I^1(f) - \kappa U_I^1(\delta_0) - (U_I^2(f) - \kappa U_I^2(\delta_0)) = \kappa a_f(U_I^1(I^*) - U_I^1(\delta_0)) - a_f(U_I^2(I^*) - U_I^2(\delta_0))$, which leads to

$$U_I^2(f) - \kappa U_I^1(f) = U_I^2(\delta_0) - \kappa U_I^1(\delta_0)$$

due to $U_I^2(I^*) - U_I^2(\delta_0) = \kappa(U_I^1(I^*) - U_I^1(\delta_0))$.

Next, we show there exist $\lambda \in \mathbb{R}^{|\Omega|}$ and $b \in \mathbb{R}$ such that $V_2(\pi) = \kappa V_1(\pi) + \lambda \cdot \pi + b$. It suffices to show $V_2(\pi) - \kappa V_1(\pi)$ is an affine function of π. Fix any two beliefs $\pi_1, \pi_2 \in \Delta(\Omega)$ and $\gamma \in [0,1]$, let $\pi = \gamma\pi_1 + (1-\gamma)\pi_2$. Then, there exists

a $\pi' \in \Delta(\Omega)$ and $\epsilon \in (0,1)$ such that $\epsilon\pi + (1-\epsilon)\pi' = \mu_0$ and $\pi' = \frac{\mu_0 - \epsilon\pi}{1-\epsilon}$. Take $f \in \mathcal{F}^*$ such that the induced posterior distribution for f is π with probability ϵ and π' with probability $1-\epsilon$. Take $g \in \mathcal{F}^*$ such that the induced posterior distribution for g is π_1 with probability $\gamma\epsilon$, π_2 with probability $(1-\gamma)\epsilon$ and π' with probability $1-\epsilon$. The process is feasible since there are infinite prizes between 0 and $\min_{\succsim} \text{supp}(I^*)$. Therefore, using $U_I^2(h) - \kappa U_I^1(h) = U_I^2(\delta_0) - \kappa U_I^1(\delta_0)$ for any $f \in \mathcal{F}^*$, we get $U_I^2(f) - \kappa U_I^1(f) = U_I^2(g) - \kappa U_I^1(g)$, which implies $\epsilon V_2(\pi) - \kappa\epsilon V_1(\pi) = \gamma\epsilon V_2(\pi_1) + (1-\gamma)\epsilon V_2(\pi_2) - \kappa(\epsilon V_1(\pi_1) + (1-\gamma)\epsilon V_1(\pi_2))$ (due to $U_I^i(f) = \epsilon V_2(\pi) + (1-\epsilon)V_2(\pi')$ and $U_I^i(g) = \gamma\epsilon V_2(\pi_1) + (1-\gamma)\epsilon V_2(\pi') + (1-\epsilon)V_2(\pi')$ for $i \in \{1,2\}$) and then $V_2(\gamma\pi_1 + (1-\gamma)\pi_2) - \kappa\epsilon V_1(\gamma\pi_1 + (1-\gamma)\pi_2) = \gamma(V_2(\pi_1) - \kappa V_1(\pi)) + (1-\gamma)(V_2(\pi_2) - \kappa V_1(\pi_2))$ .By the arbitrariness of π_1, π_2 and γ, $V_2(\cdot) - \kappa V_1(\cdot)$ is an affine function over $\Delta(\Omega)$. So there exist $\lambda \in \mathbb{R}^{|\Omega|}$ and $b \in \mathbb{R}$ such that $V_2(\pi) = \kappa V_1(\pi) + \lambda \cdot \pi + b$.

A.4 Proof of Proposition 2

Only if: Suppose $\succsim$ admits IC representations $(\mu_1, u_1, \alpha_1, A_1)$ and $(\mu_2, u_2, \alpha_2, A_2)$. Then by theorem 2, it also admits IC-V representations (μ_1, u_1, V_1^*) and (μ_2, u_2, V_2^*), where V_i^* satisfy Equation 3.1. Therefore, by proposition 1, there exist $\kappa \in \mathbb{R}_{++}$, $\lambda \in \mathbb{R}^{|\Omega|}$, and $a, b \in \mathbb{R}$ such that $u_2 = \kappa u_1 + a$ and $V_2^*(\pi) = \kappa V_1^*(\pi) + \lambda \cdot \pi + b$ for all $\pi \in \Delta(\Omega)$.

If: Suppose $\succsim$ admits an IC representation $(\mu_1, u_1, \alpha_1, A_1)$. By theorem 2, it admits an IC-V representation (μ_1, u_1, V_1^*) where V_1^* is Lipschitz continuous. Consider $\mu_2 = \mu_1$, $u_2 = \kappa u_1 + a$ and $V_2^*(\pi) = \kappa V_1^*(\pi) + \lambda \cdot \pi + b$ for all $\pi \in \Delta(\Omega)$ where $\kappa \in \mathbb{R}_{++}$, $\lambda \in \mathbb{R}^{|\Omega|}$, and $a, b \in \mathbb{R}$. Note that the affine operation guarantees that $V_2^*(\pi)$ is also Lipschitz continuous. By proposition 1, $\succsim$ admits an IC-V representation (μ_2, u_2, V_2^*). By theorem 2, $\succsim$ admits an IC representation $(\mu_2, u_2, \alpha_2, A_2)$.

A.5 Proof of Proposition 3

Given the qualifying IC representation (μ, u, α, A), we first note that A is finite so the sup in $U_I(f)$ is attained (i.e., it is a max). Dividing $\alpha U_C(f) + (1-\alpha)U_I(f)$ by α gives $U(f) = U_C(f) + \frac{1-\alpha}{\alpha} U_I(f)$. Then, the continuation value $U_I(f)$ can be

simplified:

$$
\begin{aligned}
&U_I(f)\\
&= \sum_{x\in\text{supp}(f)} \sum_{\omega\in\Omega} \mu(\omega) f(x\mid\omega) \max_{a_{\iota,\omega}\in A} \left[\mu_{x,f}(\omega) \underbrace{u\left(u^{-1}\left(\frac{\alpha}{1-\alpha}u(\iota)\right)\right)}_{\frac{\alpha}{1-\alpha}u(\iota)} + \sum_{\omega'\neq\omega} \mu_{x,f}(\omega') \underbrace{u(0)}_{\text{normalized to 0}} \right]\\
&= \sum_{x\in\text{supp}(f)} \sum_{\omega\in\Omega} \mu(\omega) f(x\mid\omega) \max_{\omega\in\Omega} \mu_{x,f}(\omega) \frac{\alpha}{1-\alpha}u(\iota)\\
&= \sum_{x\in\text{supp}(f)} \sum_{\omega\in\Omega} \mu(\omega) f(x\mid\omega) \max_{\omega\in\Omega} \underbrace{\frac{\mu(\omega) f(x\mid\omega)}{\sum_{\omega'\in\Omega}\mu(\omega') f(x\mid\omega')}}_{\text{Bayes rule}} \frac{\alpha}{1-\alpha}u(\iota)\\
&\sum_{x\in\text{supp}(f)} \underbrace{\frac{\sum_{\omega\in\Omega}\mu(\omega) f(x\mid\omega)}{\sum_{\omega'\in\Omega}\mu(\omega') f(x\mid\omega')}}_{1} \max_{\omega\in\Omega}\mu(\omega) f(x\mid\omega) \frac{\alpha}{1-\alpha}u(\iota)\\
&= \frac{\alpha}{1-\alpha}u(\iota) \underbrace{\sum_{x\in X} \max_{\omega\in\Omega}\mu(\omega) f(x\mid\omega)}_{\mathbb{I}(f,\mu)\text{ by definition}}.
\end{aligned}
$$

So

$$U(f) = U_C(f) + \frac{1-\alpha}{\alpha}U_I(f) = U_C(f) + u(\iota)\,\mathbb{I}(f,\mu).$$

A.6 Proof of Proposition 5

If: Take μ as given. Let $U_C(f;\rho) := \sum_{\omega\in\Omega}\mu(\omega)\sum_{x\in X} f(x\mid\omega)\,u_\rho(x)$ where $u_\rho(x) = 1-e^{-\rho x}$ and let $U(f;\rho,\iota) = U_C(f;\rho) + u_\rho(\iota)\,\mathbb{I}(f,\mu)$. Note that $U(f;\rho,0) = U_C(f;\rho)$. The prerequisite and proposition 3 together give $U_C(f;\rho_1) - U_C(g;\rho_1) < 0$ and $U_C(f;\rho_2) - U_C(g;\rho_2) > 0$. Take any $\epsilon \in (U_C(f;\rho_1) - U_C(g;\rho_1), 0)$ such that $|\epsilon| < \frac{1}{2}[\mathbb{I}(f,\mu) - \mathbb{I}(g,\mu)]$. Since $U_C(f;\rho) - U_C(g;\rho)$ is continuous in ρ, there exists $\bar\rho$ such that $U_C(f;\bar\rho) - U_C(g;\bar\rho) = \epsilon$ (Intermediate Value Theorem). Fix $\bar\rho$ Let $\iota_1 = 0$, then $U(f;\bar\rho,\iota_1) - U(g;\bar\rho,\iota_1) = \epsilon < 0$, so $g \succ_{(\mu,\bar\rho,\iota_1)} f$ as desired. Since $u_{\bar\rho}(\iota)$ is continuous and strictly increasing in ι, $u_{\bar\rho}(0) = 0$, and $\lim_{\iota\to\infty} u_{\bar\rho}(\iota) = 1$, there exists $\iota_2 > 0$ such that $u_{\bar\rho}(\iota_2) = \frac{1}{2}$ (Intermediate Value Theorem). Fix ι_2. Observe that $U(f;\bar\rho,\iota_2) - U(g;\bar\rho,\iota_2) = [U_C(f;\bar\rho) - U_C(g;\bar\rho)] + \frac{1}{2}[\mathbb{I}(f,\mu) - \mathbb{I}(g,\mu)] = \epsilon + \frac{1}{2}[\mathbb{I}(f,\mu) - \mathbb{I}(g,\mu)] > 0$, so $f \succ_{(\mu,\bar\rho,\iota_2)} g$ as desired.

Only if: Take μ as given. Inherit $U_C(f;\rho)$ and $U(f;\rho,\iota)$ from the "if" proof above. Due to proposition 3, $[U(f;\bar{\rho},\iota_2) - U(g;\bar{\rho},\iota_2)] - [U(f;\bar{\rho},\iota_1) - U(g;\bar{\rho},\iota_1)] = [u_{\bar{\rho}}(\iota_2) - u_{\bar{\rho}}(\iota_1)][\mathbb{I}(f,\mu) - \mathbb{I}(g,\mu)] > 0$. Since $u_{\bar{\rho}}$ is strictly increasing, $\iota_2 > \iota_1$.